

MA 355 Winter 0809: Week 5 Problems

1. Suppose that a sequence a_1, a_2, a_3, \dots satisfies

$$0 < a_n \leq a_{2n} + a_{2n+1}$$

for all $n \geq 1$.

Prove that the series $\sum_{n=1}^{\infty} a_n$ diverges.

2. Without using any technology or integral tables, evaluate the improper integral

$$\int_0^{\pi/2} \ln(\sin x) dx.$$

Justify all steps.

3. Let \mathbf{A} and \mathbf{B} be different $n \times n$ matrices with real entries, and suppose that $\mathbf{A}^3 = \mathbf{B}^3$ and $\mathbf{A}^2\mathbf{B} = \mathbf{B}^2\mathbf{A}$.

Determine, with proof, whether $\mathbf{A}^2 + \mathbf{B}^2$ can be invertible.

4. Find all positive integer solutions (x, n) of $x^2 + 8 = 3^n$.

5. Let a_1, a_2, a_3 (where $a_1 < a_2 < a_3$) be the three real zeros of a cubic polynomial, and let c (where $a_1 < c < a_2$) be the critical value guaranteed by Rolle's Theorem. Prove that c is closer to a_1 than to a_2 .