FOL: Well-formed Formulas (wffs)

Simple terms:

1. Constants (also called simple terms): a, b, c, …
2. An infinite number of variables, any of t, u, v, w, x, y, and z, with or without numerical subscripts.

Complex terms:

3. Complex terms are formed by putting a function symbol of arity n in front of n terms (simple or complex).
4. Complex terms are used just like names (simple terms) in forming atomic wffs.

Atomic wffs:

5. Predicates (capitalized first letter, and specific arity): Cube(x), Larger(x, y)
6. Atomic wffs are formed by putting a predicate of arity n in front of n terms (enclosed in parentheses and separated by commas).
7. Atomic wffs can be built from the identity predicate, =, using infix notation: the arguments are placed on either side of the predicate.

Complex wffs:

8. If P is a wff, so is ¬P.
9. If P_1, …, P_n are wffs, so is (P_1 ^ … ^ P_n).
10. If P_1, …, P_n are wffs, so is (P_1 v … v P_n).
11. If P and Q are wffs, so is (P → Q).
12. If P and Q are wffs, so is (P ↔ Q).
13. If P is a wff and *v* is a variable (i.e., one of t, u, v, w, x, . . .), then ∀*v* P is a wff, and any occurrence of *v* in ∀*v* P is said to be bound.
14. If P is a wff and *v* is a variable (i.e., one of t, u, v, w, x, . . .), then ∃*v* P is a wff, and any occurrence of *v* in ∃*v* P is said to be bound.

Sentences:

15. A sentence is a wff in which no variables occur free (unbound).

By convention, we allow the outermost parentheses in a wff to be dropped, writing A ^ B rather than (A ^ B), but only if the parentheses enclose the whole wff.