Translating into Propositional Logic

Conditionals in English

1. **If** Max is home **then** Claire is at the library.
2. Claire is at the library **if** Max is home.
3. Max is home **if** Claire is at the library.
4. Max is home **only if** Claire is at the library.
5. Claire is at the library **provided** Max is home.
6. Max is home **if and only if** Claire is at the library.
7. Max is **not** home **nor** is Claire at the library.
8. Max is home **unless** Claire is at the library.
9. Max is home **even though** Claire is at the library.
10. Max is home **in spite of the fact** that Claire is at the library.
11. Max is home **just in case** Claire is at the library.
12. Max is home **whenever** Claire is at the library.
13. Max is home **because** Claire is at the library.

Material Conditional

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

The sentence P → Q is false in only one case: if the antecedent P is true and the consequent Q is false. Otherwise, it is true.

Sufficient Condition

In English, the expression **if** introduces what is called a *sufficient condition*, one that guarantees that something else (in this case, passing the course) will obtain.

Because of this an English sentence P *if* Q must be translated as Q → P. The sentence rules out Q being true (turning in the homework) and P being false (failing the course).

Necessary Condition

In English, the expression **only if** introduces what is called a *necessary condition*, a condition that must hold in order for something else to obtain.

For example, suppose your instructor announces at the beginning of the course that you will pass the course only if you turn in all the homework assignments. Your instructor is telling you that turning in the homework is a necessary condition for passing: if you don't do it, you won't pass.

But the instructor is not guaranteeing that you will pass if you do turn in the homework: clearly, there are other ways to fail, such as skipping the tests and getting all the homework problems wrong.

Necessary Condition

The assertion that you will pass only if you turn in all the homework really excludes just one possibility: that you pass but did not turn in all the homework. In other words, P *only if* Q is false only when P is true and Q is false, and this is just the case in which P → Q is false.

Contrast this with the assertion that you will pass the course if you turn in all the homework. Now this is a very different kettle of fish. An instructor who makes this promise is establishing a very lax grading policy: just turn in the homework and you'll get a passing grade, regardless of how well you do on the homework or whether you even bother to take the tests!

Material Condition

The following English constructions are all translated P → Q:

- If P then Q
- Q if P
- P only if Q
- Provided P, Q.

The following English constructions are all translated ¬P → Q:

- Unless P, Q
- Q unless P