# Conditionals

# Assessing the validity of an argument

1. Understand what the sentences are saying.
2. Decide whether you think the conclusion follows from the premises.
3. If you think it does not follow, or are not sure, try to find a counterexample.
4. If you think it does follow, try to give an informal proof.
5. If a formal proof is called for, use the informal proof to guide you in finding one.
6. In giving consequence proofs, both formal and informal, don't forget the tactic of working backwards.
7. In working backwards, though, always check that your intermediate goals are consequences of the available information.

# Proof 6.28

| Cube(c) v Small(c)
| Dodec(c)
|—————
| Small(c)

# Proof 6.30 EZ

¬(¬Cube(a) ^ Cube(b))

¬(¬Cube(b) v Cube(c))

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Cube(a)

# Truth-functional Completeness

- A set of connectives is *truth-functionally complete* if the connectives allow us to express every truth function.
- 16 functions of 2 terms
- Examples:
  - Nand
  - Nor
  - {^, ¬}
  - {v, ¬}
  - {v, ^, ¬}

# Methods of proof

Let P and Q be any sentences in FOL.

1. Modus ponens: From $P \rightarrow Q$ and P, infer Q
2. Biconditional elimination: From P and either $P \leftrightarrow Q$ or $Q \leftrightarrow P$, infer Q.
3. Contraposition (Modus Tolens): From $P \rightarrow Q$ and $\neg Q$, infer $\neg P$.

# Class exercise

The unicorn, if horned, is elusive and dangerous.

If elusive or mythical, the unicorn is rare.

If a mammal, the unicorn is not rare.

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The unicorn, if horned, is not a mammal.

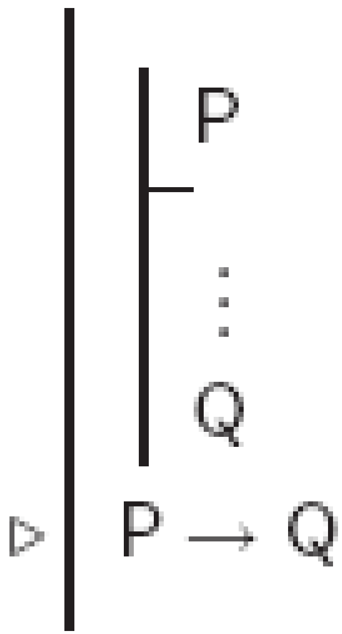
# Conditional Elimination

**Conditional Elimination ($\rightarrow$ Elim):**

$$
\begin{array}{l|l}
 & P \rightarrow Q \\
 & \vdots \\
 & P \\
 & \vdots \\
\triangleright & Q
\end{array}
$$

# Conditional Introduction

**Conditional Introduction ($\rightarrow$ Intro):**



# Biconditional Elimination

**Biconditional Elimination ($\leftrightarrow$ Elim):**

$$
\begin{array}{l|l}
 & \mathsf{P} \leftrightarrow \mathsf{Q} \text{ (or } \mathsf{Q} \leftrightarrow \mathsf{P}) \\
 & \vdots \\
 & \mathsf{P} \\
 & \vdots \\
\triangleright & \mathsf{Q}
\end{array}
$$

# Biconditional Introduction

**Biconditional Introduction ($\leftrightarrow$ Intro):**

$$
\begin{array}{l|l}
 & \begin{array}{|l} P \\ \hline \vdots \\ Q \end{array} \\
 & \begin{array}{|l} Q \\ \hline \vdots \\ P \end{array} \\
\triangleright & P \leftrightarrow Q
\end{array}
$$

# Class Exercise

$Cube(a) \vee (Cube(b) \rightarrow Tet(c))$

$Tet(c) \rightarrow Small(c)$

$(Cube(b) \rightarrow Small(c)) \rightarrow Small(b)$

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$\neg Cube(a) \rightarrow Small(b)$