Methods of Proof for Boolean Logic

Conjunction Elimination

Conjunction Elimination ($\wedge$ Elim):

$$\begin{array}{l|l} & \mathsf{P}_1 \wedge \ldots \wedge \mathsf{P}_i \wedge \ldots \wedge \mathsf{P}_n \\ & \vdots \\ \triangleright & \mathsf{P}_i \end{array}$$

Conjunction Introduction

Conjunction Introduction ($\wedge$ Intro):

$$\begin{array}{l|l} & P_1 \\ & \Downarrow \\ & P_n \\ & \vdots \\ \triangleright & P_1 \wedge \ldots \wedge P_n \end{array}$$

In this rule, we have used the notation:

$$\begin{array}{c} P_1 \\ \Downarrow \\ P_n \end{array}$$

to indicate that each of P_1 through P_n must appear in the proof before you can assert their conjunction. The order in which they appear does not matter,

Disjunction Introduction

Disjunction Introduction ($\vee$ Intro):

$$
\begin{array}{l|l}
 & P_i \\
 & \vdots \\
\triangleright & P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n
\end{array}
$$

Disjunction Elimination

Disjunction Elimination ($\vee$ Elim):

$P_1 \vee \ldots \vee P_n$

⋮

P_1

⋮

S

⇓

P_n

⋮

S

⋮

▷ S

Disjunction Elimination

- Proof by cases

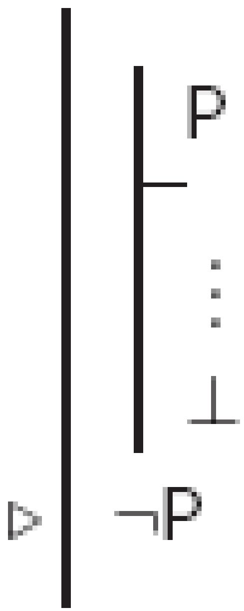
Negation Elimination

Negation Elimination (¬ Elim):

$$
\begin{array}{l|l}
 & \neg\neg P \\
 & \vdots \\
\triangleright & P
\end{array}
$$

Negation Introduction

Negation Introduction (¬ Intro):



Negation Introduction

- Proof by contradiction

⊥ Introduction

⊥ Introduction (⊥ Intro):

$$
\begin{array}{l|l}
 & P \\
 & \vdots \\
 & \neg P \\
 & \vdots \\
\triangleright & \bot
\end{array}
$$

⊥ Elimination

⊥ Elimination (⊥ Elim):

$$\bot$$

$$\vdots$$

$$\rhd \; P$$

⊥ Elimination

- Proof from inconsistent premises