Tauts

- Tautology
- Tautological equivalence
- Tautological consequence

Explanation based on Truth Tables

- Tautology:

  Only has T's under main connective
- Tautological equivalence:

  Two sentences have the same truth values in a truth table
- Tautological consequence:

  Q is a tautological consequence of P, if Q has a T wherever P has one.

Logs

- Logical truth
- Logical equivalence
- Logical consequence

Logical equivalence

- If tautological equivalent then logically equivalent.
- Reverse not necessarily true, consider:

  a = b ^ Cube(a)

  a = b ^ Cube(b)

$a = b$	Cube(a)	Cube(b)	$a = b \land$ Cube(a)	$a = b \land$ Cube(b)
T	T	T	**T**	**T**
T	T	F	**T**	**F**
T	F	T	**F**	**T**
T	F	F	**F**	**F**

Proof

- Suppose: a = b ^ Cube(a) is true.
- Then both a = b and Cube(a) are true.
- By indiscernibility of identicals, we know that Cube(b) is true.
- The truth of a = b ^ Cube(a) logically implies the truth of a = b ^ Cube(b)
- The reverse holds as well.

Logical consequence

- Every tautological consequence is also a logical consequence.
- Reverse not necessarily true, consider:

  a = c which is a logical consequence of

  (a = b ^ b = c)
- There is a row in the TT in which (a = b ^ b = c) is T but in which a = c is F
- This row does not respect the meanings of the atomic sentences

Logical truth

- Every tautology is a logical truth.
- Reverse is not necessarily true, consider:

$$\neg(Larger(a,b) \wedge Larger(b,a))$$

- Sentence cannot possibly be false.
- However, there is an F in the truth table for it.

Two more defintions

- **Logically possible.** Anything that can be true on logical grounds.

  Example: Going faster than the speed of light.
- **Logically necessary.** If it is true in every logically possible circumstance.

  Examples: Any tautology. Any sentence that can be proven with no premises at all.
- **Logically necessary** and **Logical truth** are equivalent.

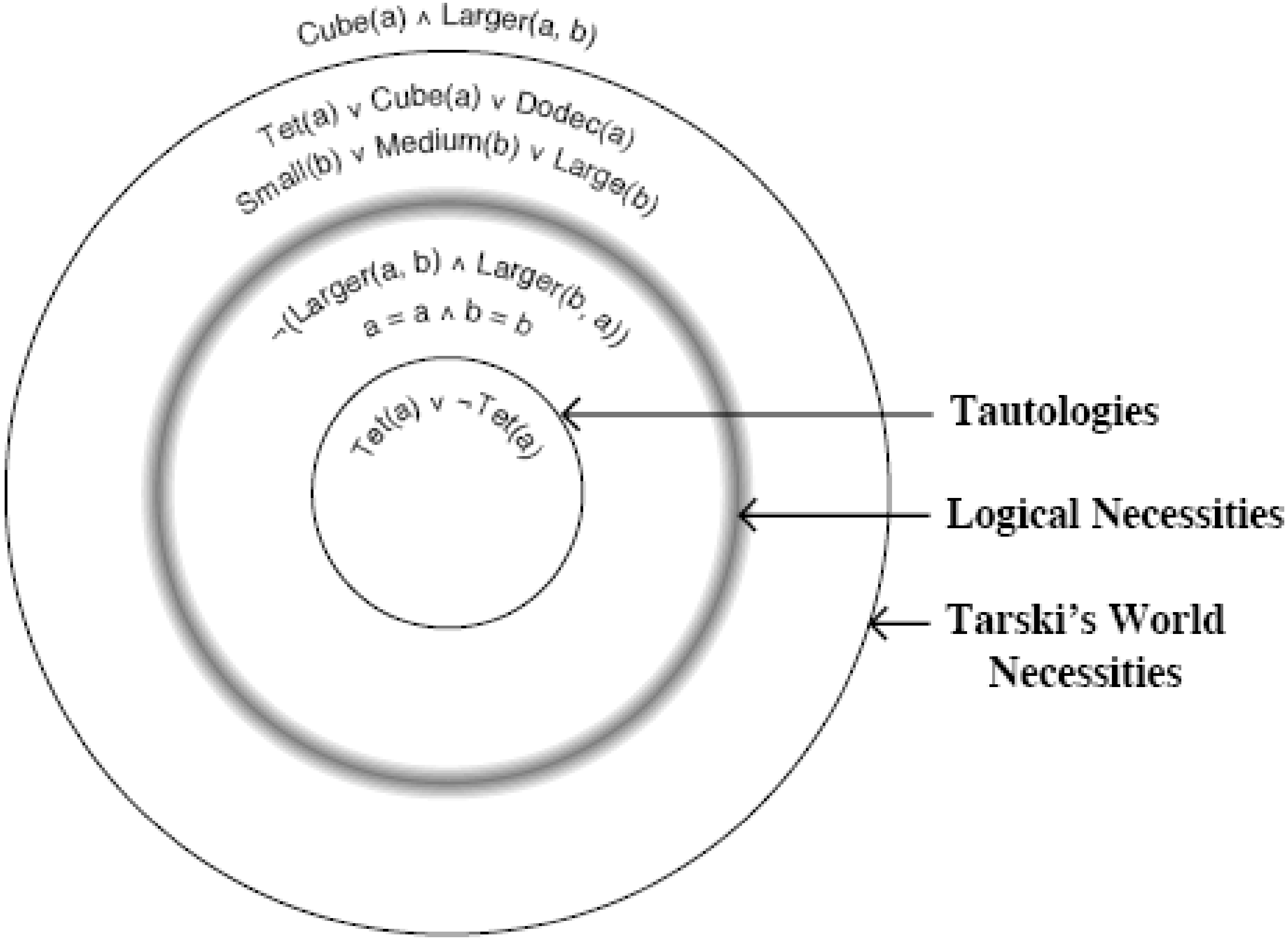

Figure 4.1: The relation between tautologies, logical truths, and TW-necessities.