

(Associativity of \wedge) An FOL sentence $P \wedge (Q \wedge R)$ is logically equivalent to $(P \wedge Q) \wedge R$, which is in turn equivalent to $P \wedge Q \wedge R$. That is,

$$P \wedge (Q \wedge R) \leftrightarrow (P \wedge Q) \wedge R \leftrightarrow P \wedge Q \wedge R$$

(Associativity of \vee) An FOL sentence $P \vee (Q \vee R)$ is logically equivalent to $(P \vee Q) \vee R$, which is in turn equivalent to $P \vee Q \vee R$. That is,

$$P \vee (Q \vee R) \leftrightarrow (P \vee Q) \vee R \leftrightarrow P \vee Q \vee R$$

(Commutativity of \wedge) A conjunction $P \wedge Q$ is logically equivalent to $Q \wedge P$. That is,

$$P \wedge Q \leftrightarrow Q \wedge P$$

As a result, any rearrangement of the conjuncts of an FOL sentence is logically equivalent to the original. For example, $P \wedge Q \wedge R$ is equivalent to $R \wedge Q \wedge P$.

(Commutativity of \vee) A disjunction $P \vee Q$ is logically equivalent to $Q \vee P$. That is,

$$P \vee Q \leftrightarrow Q \vee P$$

As a result, any rearrangement of the disjuncts of an FOL sentence is logically equivalent to the original. For example, $P \vee Q \vee R$ is equivalent to $R \vee Q \vee P$.

(Idempotence of \wedge) A conjunction $P \wedge P$ is equivalent to P . That is,

$$P \wedge P \leftrightarrow P$$

More generally (given Commutativity), any conjunction with a repeated conjunct is equivalent to the result of removing all but one occurrence of that conjunct. For example, $P \wedge Q \wedge P$ is equivalent to $P \wedge Q$.

(Idempotence of \vee) A disjunction $P \vee P$ is equivalent to P . That is,

$$P \vee P \leftrightarrow P$$

More generally (given Commutativity), any disjunction with a repeated disjunct is equivalent to the result of removing all but one occurrence of that disjunct. For example, $P \vee Q \vee P$ is equivalent to $P \vee Q$.

(Distribution Laws) For any sentence P , Q , and R :

$$\begin{aligned} P \wedge (Q \vee R) &\leftrightarrow (P \wedge Q) \vee (P \wedge R) \\ P \vee (Q \wedge R) &\leftrightarrow (P \vee Q) \wedge (P \vee R) \end{aligned}$$

(De Morgan's Laws) For any sentences P and Q :

$$\begin{aligned} \neg\neg P &\leftrightarrow P \\ \neg(P \wedge Q) &\leftrightarrow (\neg P \vee \neg Q) \\ \neg(P \vee Q) &\leftrightarrow (\neg P \wedge \neg Q) \end{aligned}$$