The Logic of Atomic Sentences

Valid and Sound Arguments

- An *argument* is a series of statements in which one, called the *conclusion*, is meant to be a consequence of the others, called the *premises.*

- An argument is *valid* if the conclusion must be true in any circumstance in which the premises are true. We say that the conclusion of a logically valid arguments is a *logical consequence* of its premises.

- An argument is *sound* if it is valid and the premises are true.

Methods of Proof

- A proof of a statement **S** from premises **P$_1$, …., P$_n$** is a step-by-step demonstration which shows that **S** *must* be true in any circumstances in which the premises **P$_1$, …, P$_n$** are all true.

- Informal and formal proofs differ in style, not in rigor.

Proofs Involving the Identity Symbol

- = Elim: If b = c, then whatever holds of b holds of c. This is also known as the **indiscernibility of identicals.**

- = Intro: Sentences of the form b = b are always true (in FOL). This is also known as the **reflexivity of identity.**

- **Symmetry of Identity:** If b = c, then c = b.

- **Transitivity of Identity:** If a = b and b = c, then a = c.

Formal Proofs

$$
\begin{array}{ll}
P & \\
Q & \\
R & \\
\hline
S_1 & \text{Justification 1} \\
\vdots & \vdots \\
S_n & \text{Justification n} \\
S & \text{Justification n+1}
\end{array}
$$

Rule: Identity Introduction (= Intro)

Identity Introduction (= Intro):

▷ | $n = n$

Identity Elimination (= Elim)

Identity Elimination (= Elim):

$$P(n)$$

$$\vdots$$

$$n = m$$

$$\vdots$$

▷ $$P(m)$$

Reiteration (Reit)

Reiteration (Reit):

$$
\begin{array}{c|l}
& P \\
& \vdots \\
\triangleright & P \\
\end{array}
$$

Fitch Style Proofs

- Demonstration of the program Fitch

Demonstrating Nonconsequence

- To demonstrate the invalidity of an argument with premises $P_1,...,P_n$ and conclusion Q, find a counterexample: a possible circumstance that makes $P_1,...,P_n$ true but Q false. Such a counterexample shows that Q is not a consequence of $P_1,...,P_n$.