Thus, four *instances* of $P[x,f(y),B]$ are:

$$P[z,f(w),B]$$
$$P[x,f(A),B]$$
$$P[g(z),f(A),B]$$
$$P[C,f(A),B]$$

The first instance is called an *alphabetic variant* of the original literal because we have merely substituted different variables for the variables appearing in $P[x,f(y),B]$. The last of the four instances shown above is called a *ground instance*, since none of the terms in the literal contains variables.

We can represent any substitution by a set of ordered pairs $s = \{t_1/v_1, t_2/v_2, \ldots, t_n/v_n\}$. The pair t_i/v_i means that term t_i is substituted for variable v_i throughout. We insist that a substitution be such that each occurrence of a variable have the same term substituted for it. Also, no variable can be replaced by a term containing that same variable. The substitutions used above in obtaining the four instances of $P[x,f(y),B]$ are:

$$s1 = \{z/x, w/y\}$$
$$s2 = \{A/y\}$$
$$s3 = \{g(z)/x, A/y\}$$
$$s4 = \{C/x, A/y\}$$

To denote a substitution instance of an expression, E, using a substitution, s, we write Es. Thus,

$$P[z,f(w),B] = P[x,f(y),B]s1 .$$

The composition of two substitutions $s1$ and $s2$ is denoted by $s1s2$, which is that substitution obtained by applying $s2$ to the terms of $s1$ and then adding any pairs of $s2$ having variables not occurring among the variables of $s1$. Thus,

$$\{g(x,y)/z\}\{A/x, B/y, C/w, D/z\} = \{g(A,B)/z, A/x, B/y, C/w\} .$$

It can be shown that applying $s1$ and $s2$ successively to an expression L is the same as applying $s1s2$ to L; that is, $(Ls1)s2 = L(s1s2)$. It can also be shown that the composition of substitutions is associative:

$$(s1s2)s3 = s1(s2s3) .$$

list-structured expressions. [The literal $P(x, f(A, y))$ is written as $(P\ x\ (f\ A\ y))$ in list-structured form.]

```
Recursive Procedure UNIFY(E1, E2)

if either E1 or E2 is an atom (that is, a
      predicate symbol, a function symbol, a
      constant symbol, a negation symbol or a variable),
      interchange the arguments E1 and E2 (if
      necessary) so that E1 is an atom, and do:
  begin
    if E1 and E2 are identical, return NIL
    if E1 is a variable, do:
      begin
        if E1 occurs in E2, return FAIL
        return {E2/E1}
      end
    if E2 is a variable, return {E1/E2}
    return FAIL
  end
F1 ← the first element of E1, T1 ← the rest of E1
F2 ← the first element of E2, T2 ← the rest of E2
Z1 ← UNIFY(F1, F2)
if Z1 = FAIL, return FAIL
G1 ← result of applying Z1 to T1
G2 ← result of applying Z1 to T2
Z2 ← UNIFY(G1, G2)
if Z2 = FAIL, return FAIL
return the composition of Z1 and Z2
```

Table 4.2
Unifiable Sets

Sets of Literals	Most General Common Substitution Instances
$\{P(x), P(A)\}$	$P(A)$
$\{P[f(x), y, g(y)], P[f(x), z, g(x)]\}$	$P[f(x), x, g(x)]$
$\{P[f(x, g(A, y)), g(A, y)], P[f(x, z), z]\}$	$P[f(x, g(A, y)), g(A, y)]$