First-Order Structures Satisfaction

Introduction

- A First-Order Structure is analogous to a truth assignment in Propositional Logic.
- For quantified sentences, we need a *domain of discourse*.
- It represents circumstances that determine the truth values of all the sentences of a language.
- It does it in a way that identity and FO Quantifiers are respected.

A Sample World

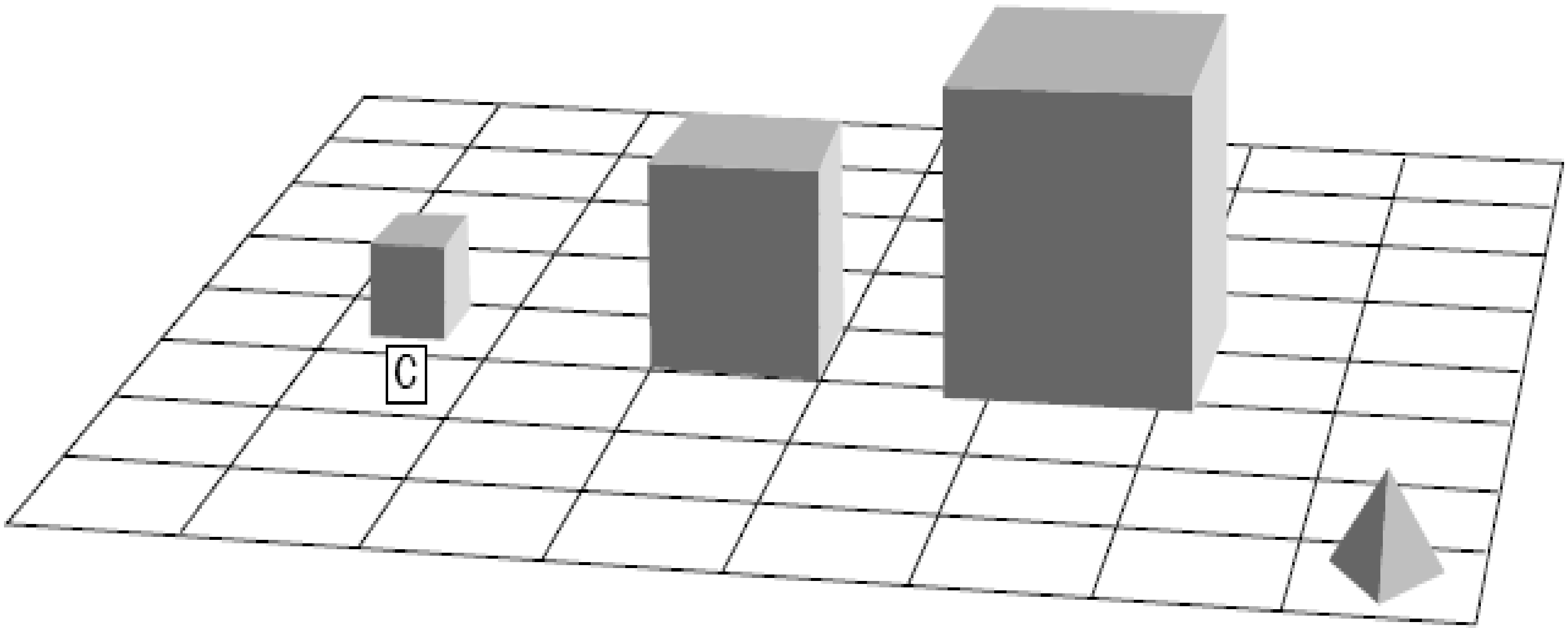

Figure 18.1: Mary Ellen's World.

Domain of Discourse

- *Domain of discourse* in this world is: $D = \{b_1, b_2, b_3, b_4\}$
- Let b_1 represent the left-most block, and go in order.
- Suppose that we have one name, **c**
- b_1 is the *referent* of the name **c**.

A Sample World

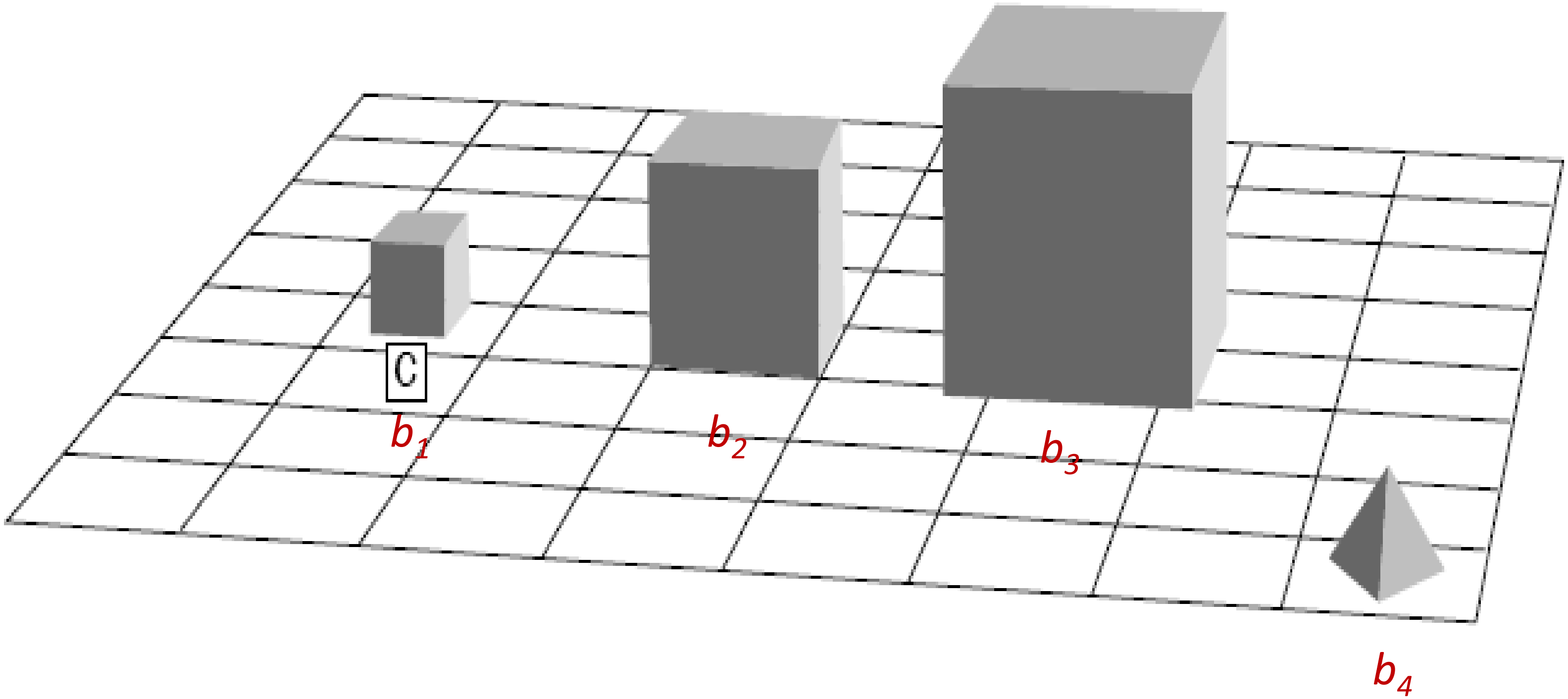

Figure 18.1: Mary Ellen's World.

Extensions

- Suppose in our language we only have the predicates: **Cube**, **Larger**, and **=**
- Assign the predicate Cube a subset of *D*:
  $Cu = \{b_1, b_2, b_3\}$
- *Cu* is said to be an *extension* of the predicate **Cube** in our structure.
- Assign the following pairs to the predicate **Larger**:
- $La = \{<b_2, b_1>, <b_3, b_1>, <b_3, b_2>, <b_2, b_4>, <b_3, b_4>\}$
- *La* is said to be an *extension* of Larger.

Extension for Identity

- Assign the following pairs to the predicate =:

$= \{<b_1, b_1>, <b_2, b_2>,<b_3, b_3>,<b_4, b_4>\}$

Models

- In order to make truth assignments, we need:
  - A domain of discourse
  - Extensions of predicates
  - Referents of constants
- We will package them into one mathematical structure, called a "model."
- A model is a function that is defined on the:
  - predicates of the language,
  - the names of the language,
  - and the quantifier symbol $\forall$.

Models

- The function $\mathfrak{M}$ is called a *first-order structure*, provided the following conditions are satisfied:

  1. $\mathfrak{M}(\forall)$ is a nonempty set *D*, called the *domain of discourse*.

  2. If **P** is an *n*-ary predicate symbol of the language, then $\mathfrak{M}$(P) is a set of *n*-tuples $<x_1, \ldots, x_n>$ of elements of *D*. This set is called the extension of **P** in $\mathfrak{M}$. It is required that the extension of the identity symbol consists of all pairs $<x, x>$, for $x \in D$.

  3. If **c** is any name of the language, then $\mathfrak{M}$(c) is an element of *D*, and is called the *referent* of c in $\mathfrak{M}$.

Modified Variable Assignments

- Consider wffs that start with a quantifier
- Let assignment *g* be defined on variable **x.**
- We want to say what it means for *g* to satisfy ∀z Likes(x, z).
- We need to be able to take any object *b* in the domain of discourse and consider the variable assignment which is just like *g* except that it assigns the value *b* to the variable **z**.
- We will say that *g* satisfies our wff ∀z Likes(x, z) if and only if every such modified assignment *g'* satisfies Likes(x, z).
- To make this a bit easier to say, we introduce the notation "g[z/b]" for the modified variable assignment.
- In general, g[v/b] is the variable assignment whose:
  - domain is that of *g* plus the variable **v**
  - which assigns the same values as *g*,
  - except that the new assignment assigns *b* to the variable **v**.

Denotation

- If a variable assignment *g* is appropriate for a wff **P**, then between $\mathfrak{M}$ and *g*, all the terms (constants and variables) in **P** have a denotation.
- For any term **t**, we write $[\![t]\!]_g^{\mathfrak{M}}$ for the denotation of t.
- Thus $[\![t]\!]_g^{\mathfrak{M}}$ is:
  - $\mathbf{t}^{\mathfrak{M}}$ if **t** is an individual constant
  - *g*(**t**) if **t** is a variable.