Truth Assignments and Truth Tables

Truth Assignments

- A *truth assignment* for a first-order language is any function *h* from the set of all atomic sentences of that language into the set {TRUE, FALSE}.
- Reminder: An atomic sentence is one that does not contain logical connectives.
- In other words, for each atomic sentence A of the language, *h* gives us a truth value, written h(A), either TRUE or FALSE.
- Examples:

  h(Mammal(fido)) = TRUE

  h(Weather(today, nice)) = TRUE

  h(Millionaire(Michael)) = FALSE

Truth Assignments and Truth Tables

- We can think of each such function *h* as representing a single row of the complete truth table for the language.

h

Truth Assignments for Complex Sentences

- Extend *h* to $\hat{h}$
- This new function is defined over the set of **all** sentences of the language.
- In other words, $\hat{h}$ fills in the values of the truth tables for all sentences of the language.

Truth Assignments for Complex Sentences

- $\hat{h}$(Q) = h(Q) for atomic sentences Q.
- $\hat{h}$(¬Q) = TRUE if and only if $\hat{h}$(Q) = FALSE;
- $\hat{h}$(Q ^ R) = TRUE if and only if $\hat{h}$(Q) = TRUE and $\hat{h}$(R) = TRUE;
- $\hat{h}$(Q v R) = TRUE if and only if $\hat{h}$(Q) = TRUE or $\hat{h}$(R) = TRUE, or both.
- $\hat{h}$(Q → R) = TRUE if and only if $\hat{h}$(Q) = FALSE or $\hat{h}$(R) = TRUE, or both.
- $\hat{h}$(Q ↔ R) = TRUE if and only if $\hat{h}$(Q) = $\hat{h}$(R).

Tautologies and Satisfiability

- **Definition**. Sentence S is a *tautology* if every truth assignment *h* has S coming out true, that is, $\hat{h}$(S) = TRUE.
- **Definition**. A sentence S is a *tautological consequence* of a set *T* of sentences provided every truth assignment that makes all the sentences in *T* true also makes S true.
- **Definition**. A sentence S is *tt-satisfiable* provided there is a truth assignment *h* such that $\hat{h}$(S) = TRUE.
- **Definition**. A set *T* of sentences is *tt-satisfiable* if there is a single assignment *h* that makes each of the sentences in *T* true.

Proposition 1

- *Proposition 1.* The sentence S is a tautological consequence of the set *T* if and only if the set *T* $\cup$ {¬S} is not tt-satisfiable.
- **Proof**. [In class]

Completeness of F_T

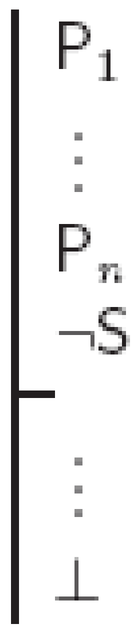
- **Theorem** (Completeness of F_T) If a sentence S is a tautological consequence of a set *T* of sentences then $T \vdash_T S$.

Lemma 2

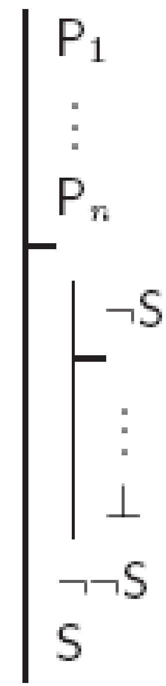
- **Lemma 2.** $T \cup \{\neg S\} \vdash_T \perp$ if and only if $T \vdash_T S$
- **Proof.** Assume $T \cup \{\neg S\} \vdash_T \perp$
- In other words, there is a proof of $\perp$ from premises $\neg S$ and certain sentences $P_1,\ldots,P_n$ of *T*.

Proof cont'd

- Arrange the premises so that:



Rearrange to:



Other direction

- **Proof.** Assume $T \vdash_T S$
- In other words, there is a proof of S from certain sentences $P_1,\ldots,P_n$ of *T*.
- You finish!