Multiple Quantifiers

Presidential Example

- *You may fool all of the people some of the time; you can even fool some of the people all of the time; but you can't fool all of the people all of the time.* [Abe Lincoln]

Multiple uses of a Single Quantifier

- What do the following two sentences mean?
  ∃x ∃y [Cube(x) ^ Tet(y) ^ LeftOf(x, y)]
  ∀x ∀y [(Cube(x) ^ Tet(y)) → LeftOf(x, y)]

Prenex Form

- *Prenex form*. All quantifiers are out in front.
- The quantifiers from the prior example are in prenex form.
- There is no need for it:
  ∃x [Cube(x) ^ ∃y (Tet(y) ^ LeftOf(x, y))]
  ∀x [Cube(x) → ∀y (Tet(y) → LeftOf(x, y))]

Translation Confirmed

P	Q	R	(P ^ Q) -> R		P -> (Q -> R)	
F	F	F	F	T	T	
F	F	T	F	T	T	
F	T	F	F	T	T	
F	T	T	F	T	T	
T	F	F	F	T	T	T
T	F	T	F	T	T	T
T	T	F	T	F	F	F
T	T	T	T	T	T	T

Distinct Variables and Objects

- Notice that in "$\forall$x $\forall$y" the same object can be assigned to x and y.
- You cannot assume that x $\neq$ y.
- In fact, $\forall$x $\forall$y P(x,y) logically implies:

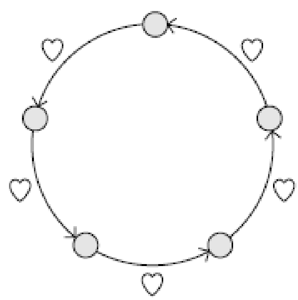
  $\forall$x P(x,x)
- $\exists$x P(x,x) logically implies:

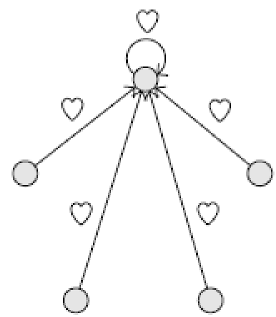
  $\exists$x $\exists$y P(x,y)

Order of Quantifiers

∀x ∃y Likes(x,y) is **not** the same as:

∃y ∀x Likes(x,y) Consider:





Order of Quantifiers

- What do the following sentences mean?

  ∀x ∃y Likes(y, x)

  ∃y ∀x Likes(y, x)

Exactly One

$\exists x\ (Cube(x) \wedge \forall y\ (Cube(y) \rightarrow y = x))$

Step-by-Step Method of Translation

- Each cube is to the left of a tetrahedron.
- Start with:

  $\forall x\ (Cube(x) \rightarrow \text{is-to-the-left-of-a-tetrahedron}(x))$
- "is-to-the-left-of-a-tetrahedron" can be formalized as:

  $\exists y\ (Tet(y) \wedge Leftof(x, y))$
- Resulting in:

  $\forall x\ (Cube(x) \rightarrow \exists y\ (Tet(y) \wedge Leftof(x, y)))$

Example

- Translate:

  No cube to the right of a tetrahedron is to the left of a larger dodecahedron.

Paraphrasing the Original Sentence

- Consider: "If a freshman takes a logic class, then he or she must be smart."
- Using step-by-step procedure:

  $\exists x(\text{Freshman}(x) \wedge \exists y(\text{LogicClass}(y) \wedge \text{Takes}(x,y))) \rightarrow \text{Smart}(x)$
- Not a sentence.

Paraphrasing the Original Sentence

- Consider: "If a freshman takes a logic class, then he or she must be smart."
- Paraphrase sentence: "Every freshman who takes a logic class must be smart."
- Using step-by-step procedure:

  ∀x[(Freshman(x) ^ ∃y(LogicClass(y) ^ Takes(x,y))) → Smart(x)]

Donkey Sentences

- Consider: "Every farmer who owns a donkey beats it."
- Step-by-step:

  ∀x(Farmer(x) ^ ∃y(Donkey(y) ^ Owns(x,y))) → Beats(x,y)
- Y occurs free in "Beats(x,y)"
- Move parentheses:

  ∀x(Farmer(x) ^ ∃y(Donkey(y) ^ Owns(x,y) ^Beats(x,y)))

Donkey Sentences

- The resulting sentence does not capture the meaning of the English sentence:

  $\forall x(Farmer(x) \wedge \exists y(Donkey(y) \wedge Owns(x,y) \wedge Beats(x,y)))$
- Instead, paraphrase the original sentence from: "Every farmer who owns a donkey beats it."
- to: "Every donkey owned by any farmer is beaten by them."

  $\forall x(Donkey(x) \rightarrow \forall y((Farmer(y) \wedge Owns(y,x)) \rightarrow Beats(y,x)))$

Ambiguity

- Every minute a man is mugged in New York City. We are going to interview him tonight.