# “Ant Colony Optimization: The Traveling Salesman Problem”

Section 2.3 from *Swarm Intelligence: From Natural to Artificial Systems*
by Bonabeau, Dorigo, and Theraulaz

# Ant Colony System (ACS)

- Improvements on Ant System made by Dorigo and Gambardella
- Based on four modifications of AS:
  - Different transition rule
  - Different pheromone trail update rule
  - Use local updates of pheromone trail
  - Use candidate list to restrict choices of next city

# Transition Rule

- Uses a tunable parameter $q_0$ to adjust the probability that an ant will explore or reinforce existing paths
- Where $q > q_0$, ACS transition rule is identical to AS
- Where $q \leq q_0$, ACS exploits knowledge available about the problem (distances, existing pheromone trails, etc.)
- Gradually increasing $q_0$ would allow the algorithm to favor exploration early and existing paths afterward

# Initialization

**Algorithm 2.2** High-level description of ACS-TSP

/* **Initialization** */
**For** every edge $(i, j)$ **do**
  $\tau_{ij}(0) = \tau_0$
**End For**
**For** $k = 1$ to $m$ **do**
  Place ant $k$ on a randomly chosen city
**End For**
**Let** $T^+$ be the shortest tour found from beginning and $L^+$ its length

# Transition Rule

/* Main loop */
**For** $t = 1$ to $t_{\max}$ **do**
**For** $k = 1$ to $m$ **do**
Build tour $T^k(t)$ by applying $n-1$ times the following steps:
**If** exists at least one city $j \in$ candidate list **then**
Choose the next city $j$, $j \in J_i^k$, among the $cl$ cities in the candidate list as follows

$$j = \begin{cases} \arg\max_{u \in J_i^k} \{[\tau_{iu}(t)] \cdot [\eta_{iu}]^\beta\} & \text{if } q \leq q_0; \\ J & \text{if } q > q_0, \end{cases}$$

where $J \in J_i^k$ is chosen according to the probability:

$$p_{ij}^k(t) = \frac{[\tau_{ij}(t)] \cdot [\eta_{ij}]^\beta}{\sum_{l \in J_i^k} [\tau_{il}(t)] \cdot [\eta_{il}]^\beta},$$

and where $i$ is the current city
**Else**
choose the closest $j \in J_i^k$

# Pheromone Trail Update Rules

- Only the ant with the best tour lays pheromones
- Encourages ants to search paths near the best found so far

# Local Pheromone Trail Updates

- Each time an ant moves from one city to another, the pheromone concentration on that edge is reduced
- This makes visited edges less attractive and encourages exploration

# Pheromone Update

**End If**
After each transition ant $k$ applies the local update rule:
$\tau_{ij}(t) \leftarrow (1-\rho)\cdot\tau_{ij}(t) + \rho\cdot\tau_0$
**End For**
**For** $k = 1$ to $m$ **do**
Compute the length $L^k(t)$ of the tour $T^k(t)$ produced by ant $k$
**End For**
**If** an improved tour is found **then**
update $T^+$ and $L^+$
**End If**
**For** every edge $(i, j) \in T^+$ **do**
Update pheromone trails by applying the rule:
$\tau_{ij}(t) \leftarrow (1-\rho)\cdot\tau_{ij}(t) + \rho\cdot\Delta\tau_{ij}(t)$ where $\Delta\tau_{ij}(t) = 1/L^+$
**End For**
**For** every edge $(i, j)$ **do**
$\tau_{ij}(t+1) = \tau_{ij}(t)$
**End For**

# Example Solution

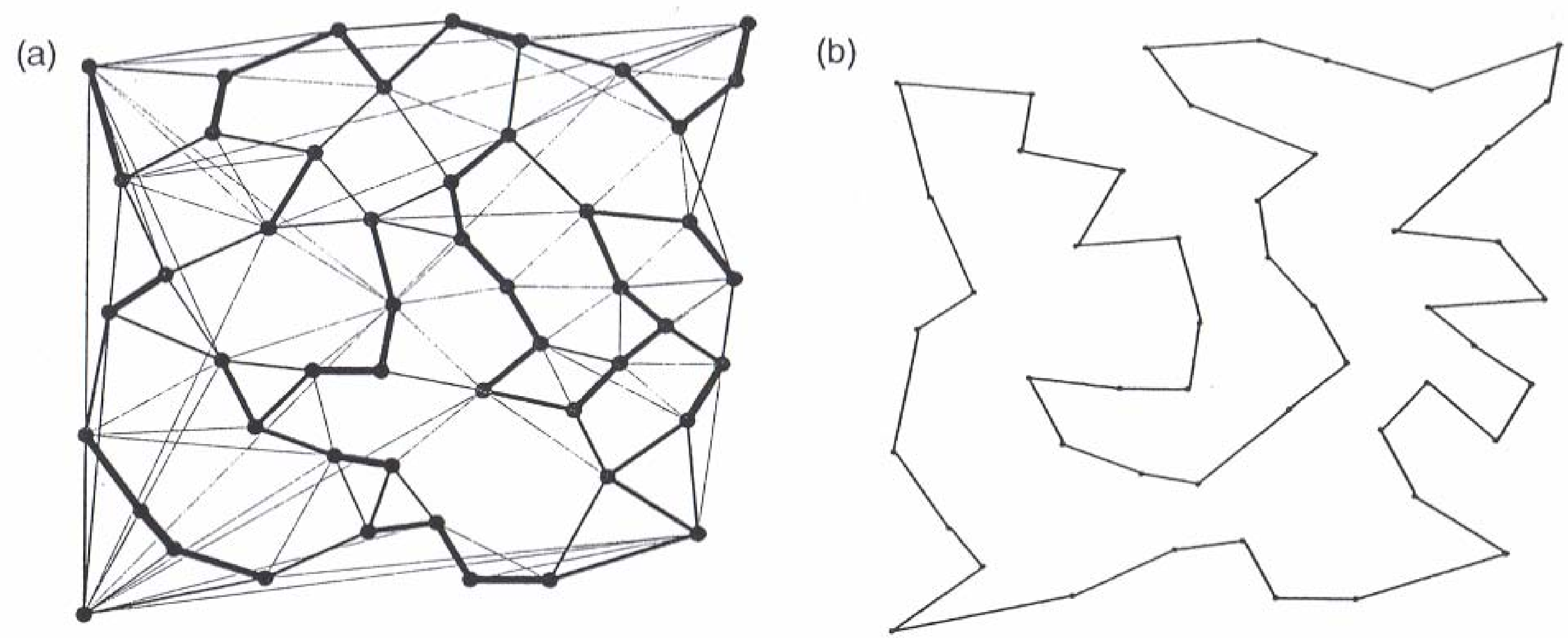


FIGURE 2.15 (a) An example of a trail configuration found by ACS in a 50-city problem (Eil50). Line thickness reflects pheromone concentration. (b) Best solution found by ants.

# ACS Conclusions

TABLE 2.2 Comparison of ACS-TSP with other algorithms on randomly generated prob lems. Results for EN are from Durbin and Willshaw [117] and results for SA and SOM ar from Potvin [262]. For ACS-TSP, averages are over 25 runs. Best results are in boldface From Dorigo and Gambardella [111].

| | ACS-TSP | SA | EN | SOM |
|---|---|---|---|---|
| City set 1 (50-city problem) | **5.88** | **5.88** | 5.98 | 6.06 |
| City set 2 (50-city problem) | 6.05 | **6.01** | 6.03 | 6.25 |
| City set 3 (50-city problem) | **5.58** | 5.65 | 5.70 | 5.83 |
| City set 4 (50-city problem) | **5.74** | 5.81 | 5.86 | 5.87 |
| City set 5 (50-city problem) | **6.18** | 6.33 | 6.49 | 6.70 |

TABLE 2.3 Comparison of ACS-TSP with GA, EP, and SA on four test problems (available at TSPLIB: http://www.iwr.uni-heidelberg.de/iwr/comopt/soft/TSPLIB95/TSPLIB.html). Results for ACS-TSP are from Dorigo and Gambardella [111]. ACS-TSP is run for 1250 iterations using 20 ants, which amounts to approximately the same number of tours searched by the other heuristics; averages are over 15 runs. Results for GA are from Bersini et al. [20] for the KroA100 problem, and from Whitley et al. [327] for the Oliver30, Eil50 and Eil75 problem; results for EP are from Fogel [125]; and results for SA from Lin et al. [222]. For each algorithm results are given in two columns: the first one gives the best integer tour length (that is, the tour length obtained when distances among cities are given as integer numbers), and, in parenthesis, the best real tour length (that is, the tour length obtained when distances among cities are given as real numbers); the second one gives the number of tours that were generated before the best integer tour length was discovered. Best results are in boldface. (N/A: Not available.)

| | ACS-TSP best | ACS-TSP # iter. | GA best | GA # iter. | EP best | EP # iter. | SA best | SA # iter. |
|---|---|---|---|---|---|---|---|---|
| Eil50 (50-city problem) | **425** (427.96) | **1830** | 428 (N/A) | 25000 | 426 (**427.86**) | 100000 | 443 (N/A) | 68512 |
| Eil75 (75-city problem) | **535** (**542.37**) | **3480** | 545 (N/A) | 80000 | 542 (549.18) | 325000 | 580 (N/A) | 173250 |
| KroA100 (100-city problem) | **21282** (**21285.44**) | **4820** | 21761 (N/A) | 103000 | N/A (N/A) | N/A | N/A (N/A) | N/A |

- ACS performs comparably or better than other TSP algorithms

# Further Enhancements

- Added 3-opt procedure to locally minimize tour lengths between ACS iterations
  - Performed comparably with STSP algorithm that won the First International Contest on Evolutionary Optimization
- Other unimplemented improvements:
  - Allow the best r ants to update the pheromone trail, instead of a single ant
  - Remove pheromone from edges of worst tours
  - More powerful local search