A Theorem Prover for a Diagrammatic Blocks World

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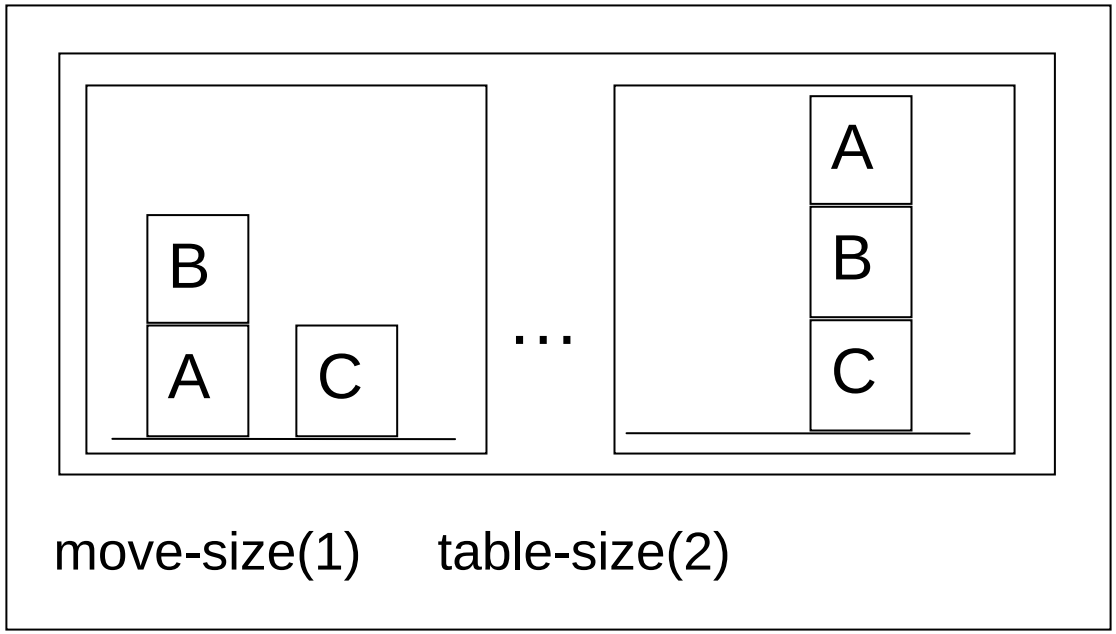
Overview

- Introduction
- Syntax of diagrams
- Types of proofs
- Sample consequence proof
- Primary rules of inference
- Implementation
- Conclusions

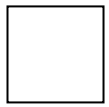
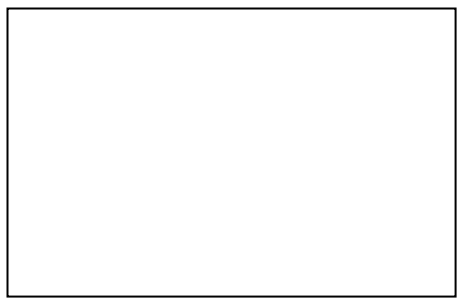
Introduction

- Developed a system in which to use diagrams to give proofs in the blocks world.
- Proven the system sound and complete
- Thereby shown that one can reason validly with diagrams
- Implemented it, showing that this reasoning can be automated

Sample Diagram



Diagrammatic Symbols

- Block Square: 
- Table line: _____
- Situation connectors: -> …
- Boundary rectangle: 

Well-Formed Frame

- Exactly one boundary rectangle
- Exactly one table line drawn horizontally near the bottom of the boundary rectangle
- Block squares are drawn on the table line or squarely on top of each other. They do not overlap with each other or the boundary rectangle.
- Each block square contains exactly one block constant in a non-overlapping fashion.
- No two block squares are labeled by the same block constant

Types of Proofs

- Consistency
- Non-consequence
- Consequence
- Inconsistency

Consistency Proofs

- Used to show that the given information is consistent
- This means that there is a plan
- Traditional planning problems fit this category best
- In traditional blocks world, a solution can always be found, but an optimal solution is desired, as such, it is more complex than a consistency proof

Consequence Proofs

- Used to show that some information follows from the given information
- Something holds for all plans
- Most complex proof
- Detailed example momentarily

Non-Consequence Proof

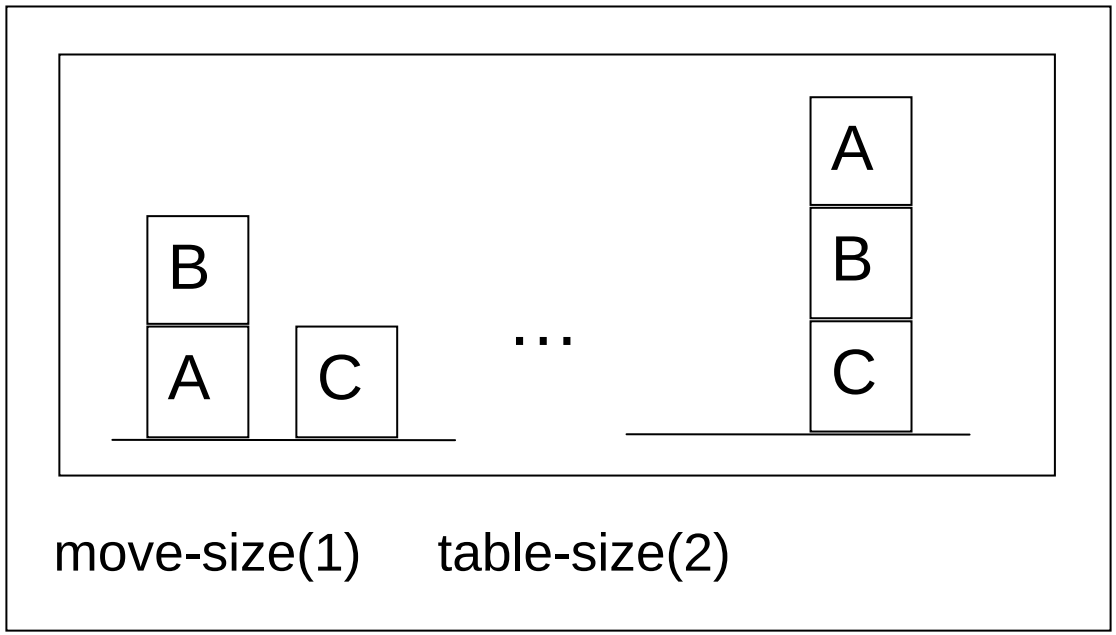
- Used to show that some information does not follow from the given information
- This means that there is an alternative plan
- With this proof, we show that a counterexample is consistent, hence, it is a consistency proof

Inconsistency Proofs

- Used to show that some information is inconsistent
- This means that there is no plan
- It is a consequence proof in which no path leads from the start state to the final state, such that it satisfies all constraints.

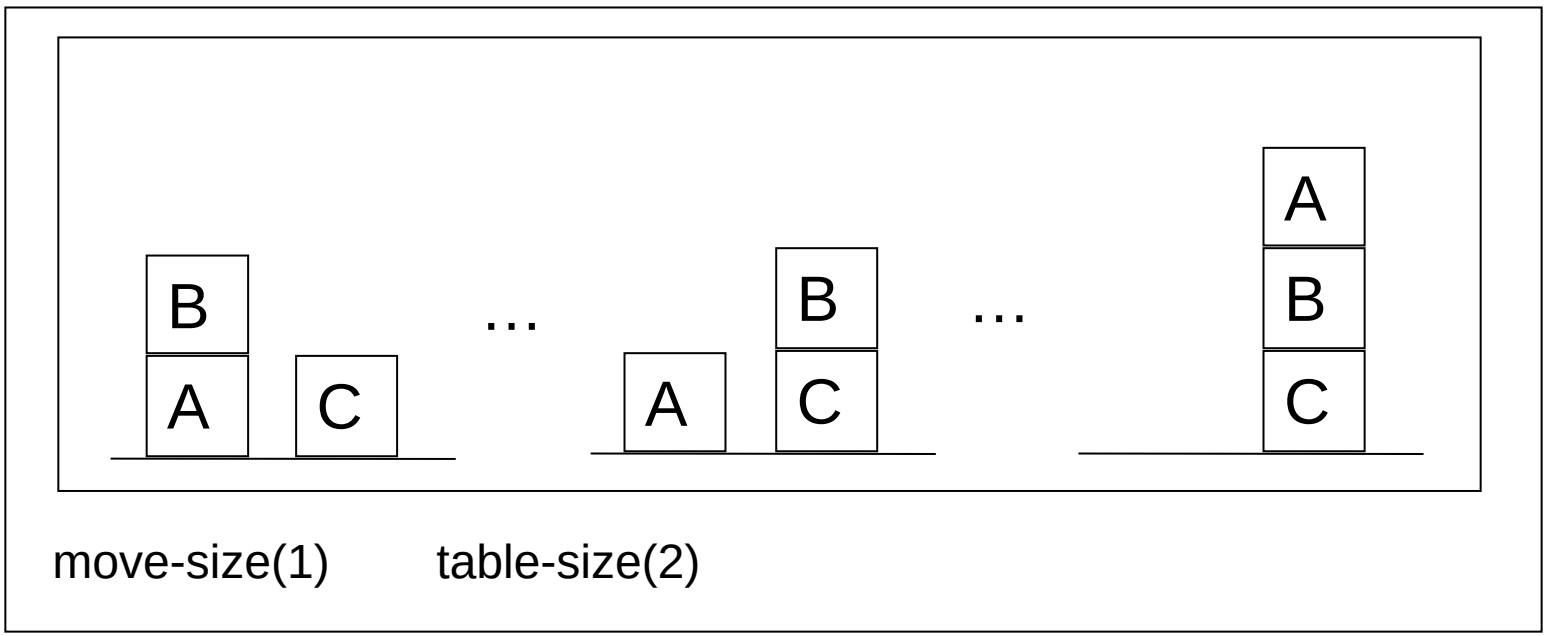
Sample Consequence Proof

- Let this diagram be the given information

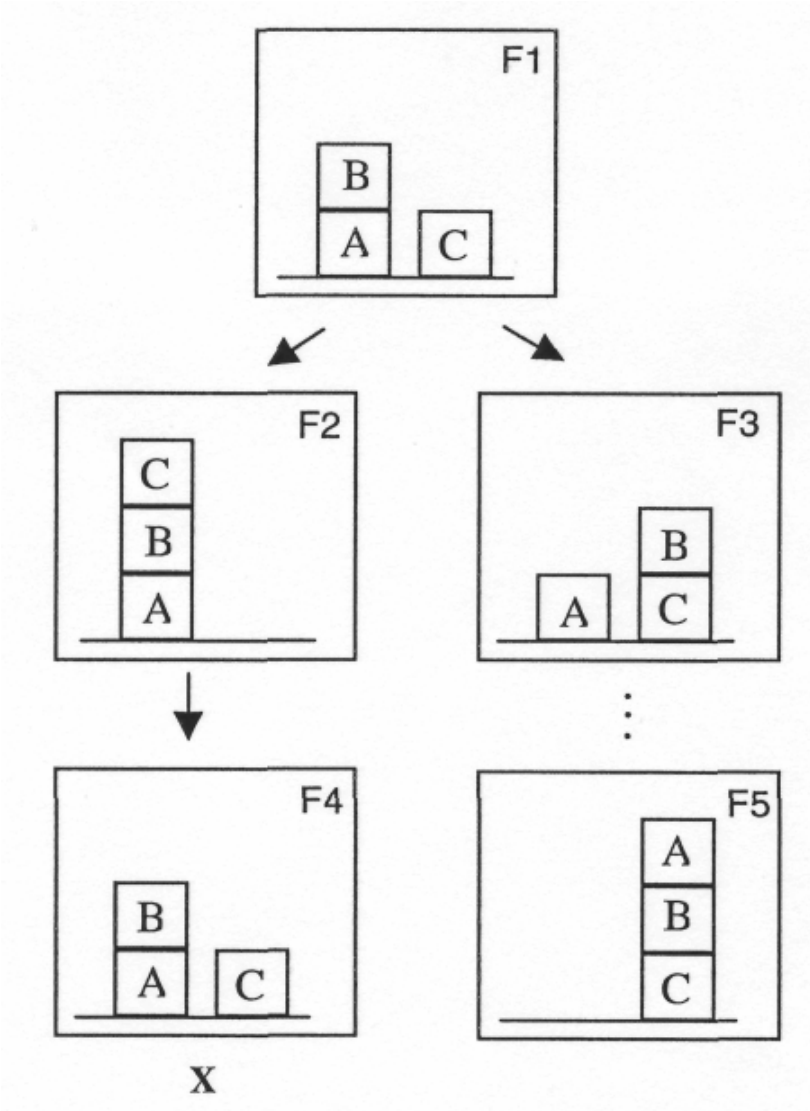


Sample Consequence Proof

- Suppose we want to show that the following information is a consequence.
- It states that in each plan we have to move B onto of C



Abbreviated Proof



Rules of Inference

- *Given*. Accept the given information

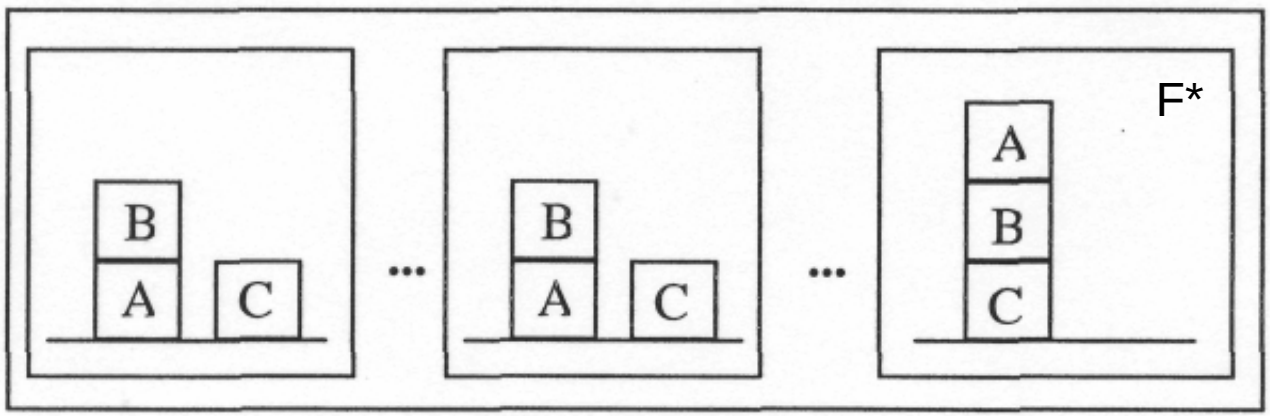
- *Move*. A frame is added, and linked by an arrow
- In the new frame, either a single block or a single tower of blocks has been redrawn

Rules of Inference

- *Cases exhausted*. This rule is used to state that diagram which were derived by the “move” rule exhaust all possible moves which can be made
- This rule takes the sentential constraints into account
- In our system, we give proofs by cases

Rules of Inference

- *Cycle.* This rule is used to make explicit that there are cycles in this domain.
- Introduce a copy of a frame, link it by triple dots and label it the last such frame before the next frame.



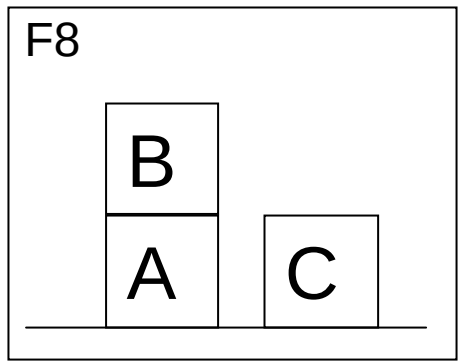
last $_{F^*}$

Rules of Inference

- *Close*. Used to put the rule “cycle” to good use.
- If a frame has been labeled “last” and an identical frame appears between it and the referenced frame, then the diagram can be closed.

Representing Diagrams

- Towers of blocks are stored as lists.
- `(define-structure (frame number info last next))`



- (make-frame ‘f8 ‘((a b) (c)) nil nil)

Implementation

- Rules of inference are a direct implementation of the ones presented.

Conclusions

- Well-formed definitions of diagrams can be given
- Rules of inference can be shown sound and complete
- Diagrams are designed for particular domains
- As such a diagrammatic system is a special-purpose system