

A water tank in the shape of the paraboloid formed when $y = x^2$ is rotated about the y -axis has a radius of five meters. The tank is initially filled with water. A hole with a radius of .2 meters is opened at the bottom of the tank at time $t = 0$. Assume that the acceleration due to gravity is a constant 9.80 m/s^2 .

1 Determine the depth of the water in the tank as a function of time.

2 The sides of the tank are perfectly insulated, so the temperature of the water follows a modified Newton's law of Cooling, in which the rate of change of the temperature is proportional to not only the difference of the temperature and the ambient temperature, but also to the exposed surface area of the water, and inversely proportional to the volume of water,

$$\frac{dT}{dt} = k \frac{SA}{V}(A - T).$$

Five minutes before the hole was opened in the tank, the temperature of the water was 40° . At the instant that the hole was opened, the temperature was 38° . If the ambient temperature is 20° , determine the temperature of the water as a function of time.

3 The water drains from the parabolic tank into another tank that initially contains 500 cubic meters of water mixed with 100 pounds of salt. At the instant that the hole is opened in the parabolic tank, draining the water into the brine, a pipe is opened causing water to flow from the second tank at a rate of 6 cubic meters per minute. Determine when there are 10 pounds of salt in the second tank.