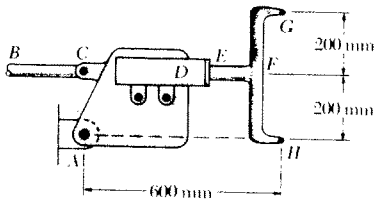
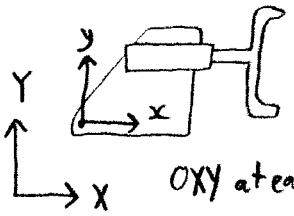
15.126 In the automated welding setup shown, the position of the two welding tips G and H is controlled by the hydraulic cylinder D and rod BC. The cylinder is bolted to the vertical plate which at the instant shown rotates counterclockwise about A with a constant angular velocity of 1.6 rad/s. Knowing that at the same instant the length EF of the welding assembly is increasing at the constant rate of 300 mm/s, determine (*a*) the velocity of tip H, (*b*) the acceleration of tip H.



Strategy: Kinematics of moving coords

Define oxy attached to plate, centered at A



a) Find $\vec{v}_H$

$$\vec{v}_H = \vec{v}_o + \vec{v}_{rel} + \vec{\omega} \times \vec{r}_{H/o}$$

$\vec{v}_o = \vec{0}$ b/c moving axis is just rotating
$\vec{v}_{rel} = 0.3\ m/s\ \hat{\imath}$
$\vec{\omega} = 1.6\ rad/s\ \hat{K}$
$\vec{r}_{H/o} = 0.6\ m\ \hat{\imath}$

$$\vec{v}_H = \vec{0} + 0.3\ m/s\ \hat{\imath} + (1.6\ rad/s\ \hat{K}) \times (0.6\ m\ \hat{\imath})$$

↖ use $\hat{\imath} = \hat{I}$ ↖ use $\hat{\imath} = \hat{I}$, so $\hat{K} \times \hat{I} = \hat{J}$

$$= 0.3\ m/s\ \hat{I} + (1.6\ rad/s)(0.6\ m)\ \hat{J}$$

$$\boxed{\vec{v}_H = 0.3\ m/s\ \hat{I} + 0.96\ m/s\ \hat{J}}$$

b) Find $\vec{a}_H$

$$\vec{a}_H = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{H/o} - \omega^2 \vec{r}_{H/o} + 2\vec{\omega} \times \vec{v}_{rel}$$

$\vec{a}_o = \vec{0}$ moving axis not accelerating
$\vec{\alpha} = \vec{0}$ moving axis at constant $\vec{\omega}$
$\vec{a}_{rel} = \vec{0}$ hydraulic cylinder moving at constant speed

$$\vec{a}_H = \vec{0} + \vec{0} + \vec{0} \times (0.6\ m\ \hat{\imath}) - (1.6\ rad/s)^2 (0.6\ m\ \hat{\imath}) + 2(1.6\ rad/s\ \hat{K}) \times (0.3\ m/s\ \hat{\imath})$$

again, use $\hat{\imath} = \hat{I}$

$$\vec{a}_H = -(1.6\ rad/s)^2 (0.6\ m\ \hat{I}) + 2(1.6\ rad/s\ \hat{K}) \times (0.3\ m/s\ \hat{I})$$

$$\boxed{\vec{a}_H = -1.536\ m/s^2\ \hat{I} + 0.96\ m/s^2\ \hat{J}}$$

The truck is moving forward at a speed of 35 mi/hr (51.33 ft/sec) and is decelerating at the rate of 10 ft/sec^2. Simultaneously, the boom is being raised and extended. At the instant considered, the angle is 30° and is increasing at the constant rate of 10 deg/sec. Also, at this instant the extension of the boom is 20 ft with an extension rate of 2 ft/sec and an extension acceleration of -1 ft/sec. Determine the global velocity and acceleration of point B.

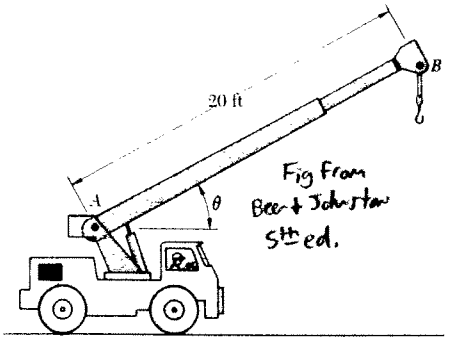
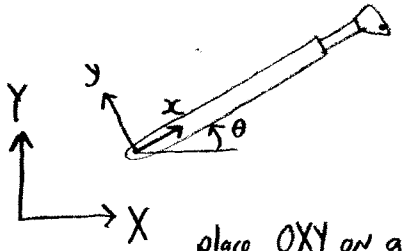


Fig from Beer + Johnston 5th ed.

Strategy = Kinematics of moving frame

Define Coord Sys

Place oxy on boom at pt A (moving coord)



place OXY on ground (fixed coord)

Thus,

$\hat{\imath} = \cos\theta\,\hat{I} + \sin\theta\,\hat{J}$

$\hat{\jmath} = -\sin\theta\,\hat{I} + \cos\theta\,\hat{J}$

a) Find $\vec{V}_B$

$\vec{V}_B = \vec{V}_o + \vec{V}_{rel} + \vec{\omega} \times \vec{r}_{B/o}$

$\vec{V}_o = 51.33\ ft/s\ \hat{I}$

$\vec{V}_{rel} = 2\ ft/s\ \hat{\imath}$

$\vec{\omega} = (10\ deg/s)\left(\frac{2\pi\ rad}{360\ deg}\right)\hat{K} = \frac{\pi}{18}\ rad/s\ \hat{K}$

$\vec{r}_{B/o} = 20\ ft\ \hat{\imath}$

} tables find each part

$\vec{V}_B = 51.33\ ft/s\ \hat{I} + 2\ ft/s\ \hat{\imath} + \left(\frac{\pi}{18}\ rad/s\ \hat{K}\right) \times (20\ ft\ \hat{\imath})$

needs coord transformation (see above)

$2\ ft/s\ \hat{\imath} = (2\ ft/s)(\cos\theta\,\hat{I} + \sin\theta\,\hat{J})$

$\left(\frac{\pi}{18}\ \frac{rad}{s}\ \hat{K}\right) \times (20\ ft\ \hat{\imath}) = \left(\frac{\pi}{18}\ \frac{rad}{s}\ \hat{K}\right) \times (20\ ft)(\cos\theta\,\hat{I} + \sin\theta\,\hat{J})$

$= \left(\frac{\pi}{18}\right)\left(20\ \frac{ft}{s}\right)(\cos\theta\,\hat{J} - \sin\theta\,\hat{I})$

$\therefore \vec{V}_B = 51.33\ ft/s\ \hat{I} + \left[2\ ft/s\ (\cos\theta\,\hat{I} + \sin\theta\,\hat{J})\right] + \left[\left(\frac{\pi}{18}\right)\left(20\ \frac{ft}{s}\right)(\cos\theta\,\hat{J} - \sin\theta\,\hat{I})\right]$

use $\theta = 30°$

$$\boxed{\vec{V}_B = 51.32\ ft/s\ \hat{I} + 4.023\ ft/s\ \hat{J}}$$

b) Find $\vec{a}_B$

$$\vec{a}_B = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{B/o} - \omega^2 \vec{r}_{B/o} + 2\vec{\omega} \times \vec{v}_{rel}$$

$\vec{a}_o = -10\ ft/s^2\ \hat{I}$

$\vec{a}_{rel} = -1\ ft/s\ \hat{\imath} \Rightarrow (-\cos\theta\ \hat{I} - \sin\theta\ \hat{J})\ ft/s$ when converted

$\vec{\alpha} = \vec{0}$ since $\vec{\omega}$ is constant

$\vec{r}_{B/o} = 20\ ft\ \hat{\imath} \Rightarrow (20ft)(\cos\theta\ \hat{I} + \sin\theta\ \hat{J})$ when converted

$\vec{\omega} = \frac{\pi}{18}\ rad/s\ \hat{K}$

$\vec{v}_{rel} = 2\ ft/s\ \hat{\imath} \Rightarrow (2\ ft/s)(\cos\theta\ \hat{I} + \sin\theta\ \hat{J})$ when converted

$$\vec{a}_B = [-10\ ft/s^2\ \hat{I}] + [-\cos\theta\ ft/s\ \hat{I} - \sin\theta\ ft/s\ \hat{J}] + [\vec{0}] - [(\tfrac{\pi}{18}\ rad/s)^2 (20ft)(\cos\theta\ \hat{I} + \sin\theta\ \hat{J})]$$
$$+ [\underbrace{2(\tfrac{\pi}{18}\ rad/s\ \hat{K}) \times (2\ ft/s)(\cos\theta\ \hat{I} + \sin\theta\ \hat{J})}_{\text{equals}}]$$
$$[(\tfrac{2\pi}{9}\ ft/s^2)(\cos\theta\ \hat{J} - \sin\theta\ \hat{I})]$$

at $\theta = 30°$

$$\vec{a}_B = [-10 - \cos 30° - (\tfrac{\pi}{18})^2 (20)(\cos 30°) - (\tfrac{2\pi}{9})(\sin 30°)]\ ft/s^2\ \hat{I}$$
$$+ [-\sin 30° - (\tfrac{\pi}{18})^2 (20)(\sin 30°) + (\tfrac{2\pi}{9})(\cos 30°)]\ ft/s^2\ \hat{J}$$

$$\boxed{\vec{a}_B = -11.74\ ft/s^2\ \hat{I} - 0.2\ ft/s^2\ \hat{J}}$$