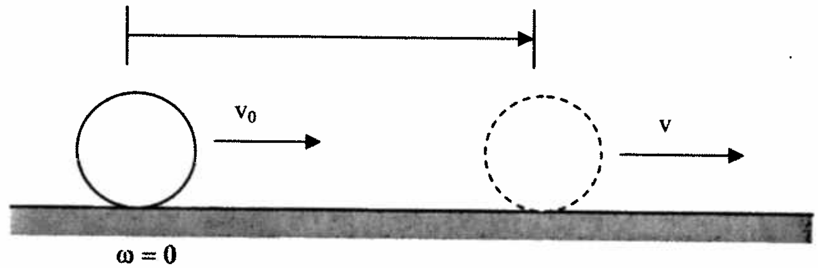
Example 4/18

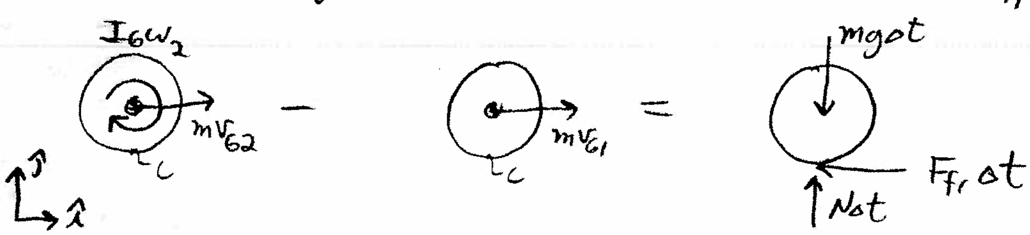
A bowling ball with radius 4.3 in. has a radius of gyration about its center of gravity of 3.28 in. If the ball is released with a velocity of 20 ft/s but with no angular velocity as it touches the alley floor, compute the time before the ball begins to roll without slipping. The coefficient of friction between the ball and the floor is 0.2.



Strategy: CLM_{FT}
CAM_{FT}
Kinematics

use $I_G = m r_G^2$ w/ r_G = radius of gyration

Draw a Momentum Diagram from just after release till roll w/o slipping



during interval it's sliding, so $F_{fr} = \mu_K N$

CLM_{FT} (sys = ball, $\hat{i}$)

$$\mathbb{P}_{sys,2x} - \mathbb{P}_{sys,1x} = \Sigma F_x \Delta t$$

① $[m V_{G2}] - [m V_{G1}] = -[\mu_K N \Delta t]$

CLM_{FT} (sys = ball, $\hat{j}$)

$$\mathbb{P}_{sys,2y} - \mathbb{P}_{sys,1y} = \Sigma F_y \Delta t$$

② $[0] - [0] = N \Delta t - m g \Delta t$

CAM_{FT} (about contact pt, sys = ball, ↻)

$$\vec{L}_{sys,c2} - \vec{L}_{sys,c1} = \Sigma \vec{M}_c \Delta t$$

③ $[I_G \omega_2 + m V_{G2} r] - [m V_{G1} r] = 0$

Kinematics at state 2, no more slipping, so $V_{G2} = \omega_2 r$ ④

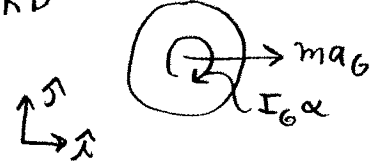
We appear to have 5 unknowns, $v_{G2}, N, \Delta t, \omega_2, m$ but mass falls out of analysis

Aside: consider the system $(2a)x = a/2 + b$, $b = 3ax^2$ } independent of a: $2x - 3x^2 = \frac{1}{2}$

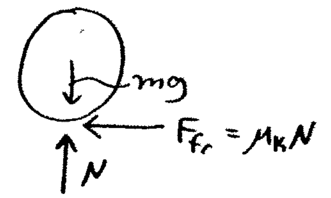
So we can solve to get $\Delta t = 1.14$ sec by hand or MAPLE

If sufficient time, solve via CAMrate + CLMrate just after release

KD



FBD



CLMrate (sys = ball, $\hat{j}$)

$$\frac{d}{dt}(\mathbb{P}_{sys,y}) = \sum F_y \Rightarrow 0 = N - mg \quad \therefore N = mg \quad (1)$$

CLMrate (sys = ball, $\hat{i}$)

$$\frac{d}{dt}(\mathbb{P}_{sys,x}) = \sum F_x \Rightarrow ma_G = -\mu_K N \quad (2)$$

CAMrate (sys = ball, $\hat{k}$, com)

$$\frac{d}{dt}(\vec{L}_{sys,com}) = \sum \vec{M}_{com} \Rightarrow I_G\alpha = \mu_K N r \quad (3)$$

Solve (2) to get $a_G = -\mu_K N/m$ then (1) to get $a_G = -\mu_K g$ constant linear accel

Solve (3) to get $\alpha = \dfrac{\mu_K N r}{m r_G^2}$

insert (1) for N $\quad \alpha = \dfrac{\mu_K \cancel{m} g r}{\cancel{m} r_G^2} = \dfrac{\mu_K g r}{r_G^2}$ constant angular accel

Constant accels, so we can use constant accel relations

$$v = v_0 + a_G t \Rightarrow v_G = v_0 + [-\mu_K g]t$$

$$\omega = \underbrace{\omega_0}_{\to 0} + \alpha t \Rightarrow \omega = \left[\frac{\mu_K g r}{r_G^2}\right]t$$

At the end of our analysis, the ball is <u>not</u> slipping, so $v_G = \omega r$ by kinematics

$$v_0 - \mu_K g t = \left[\frac{\mu_K g r t}{r_G^2}\right] r \Rightarrow t = \frac{v_0}{\left[\mu_K g\left(1 + r^2/r_G^2\right)\right]} \xrightarrow{\text{with values}} t = 1.14\,\text{sec}$$