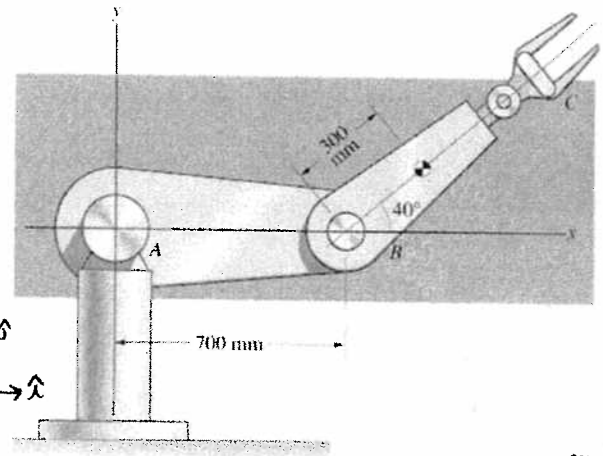
Example Problem

7.27 Arm BC has a mass of 12 kg and the mass moment of inertia about its center of mass is 3 kg-m^2. If arm AB has a constant clockwise angular velocity of 2 rad/s and arm BC has a counterclockwise angular velocity of 2 rad/s and a clockwise angular acceleration of 4 rad/s^2, determine:

a) the couple exerted on arm BC at B,
b) the reaction at B

(*taken from Dynamics, 2nd Edition by Bedford & Fowler*)



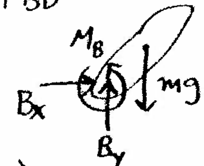
Given: $\vec{\omega}_{AB} = -2\ \text{rad/s}\ \hat{K}$ $\quad \vec{\alpha}_{AB} = 0$

$\vec{\omega}_{BC} = 2\ \text{rad/s}\ \hat{K}$ $\quad \vec{\alpha}_{BC} = -4\ \text{rad/s}^2\ \hat{K}$

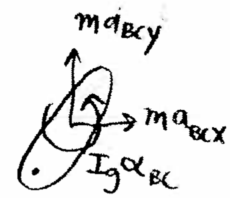
a) Find couple (Moment) exerted on arm BC at pt B

strategy: CAMrate (pt B, ⓚ, sys = BC)

FBD



KD



$$\frac{d}{dt}\left(\vec{L}_{sys,B}\right) = \sum \vec{M}_B$$

$$I_g \alpha_{BC} - m a_{BCx}(r\sin 40°) + m a_{BCy}(r\cos 40°) = M_B - mg(r\cos 40°)$$

① $$I_g \alpha_{BC} - m a_{BCx}(r\sin 40°) + m a_{BCy}(r\cos 40°) = M_B - mg(r\cos 40°)$$

$$\alpha_{BC} = -4\ \text{rad/s}^2$$

3 unknowns: a_{BCx}, a_{BCy}, M_B

CLMrate (sys = BC, î)

$$\frac{d}{dt}\left(P_{sys,x}\right) = \sum F_x$$

② $$m a_{BCx} = B_x$$ 1 new unknown: B_x

CLMrate (sys = BC, ĵ)

$$\frac{d}{dt}\left(P_{sys,y}\right) = \sum F_y$$

③ $$m a_{BCy} = B_y - mg$$ 1 new unknown: B_y

1

Strategy: Kinematics of bar AB

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$
($\vec{a}_A \to 0$, $\vec{\alpha}_{AB} \to 0$)

$$= -\omega_{AB}^2 (0.7m\,\hat{i})$$

(4) $\hat{i}$) $a_{BX} = -\omega_{AB}^2 (0.7m)$

$\hat{j}$) $a_{By} = 0$

Kinematics of bar BC

$$\vec{a}_{BC} = \vec{a}_B + \vec{\alpha}_{BC} \times \vec{r}_{GBC/B} - \omega_{BC}^2 \vec{r}_{GBC/B}$$

$$= (a_{BX}\,\hat{i}) + (\alpha_{BC}\,\hat{k}) \times (r\cos 40^\circ \hat{i} + r\sin 40^\circ \hat{j}) - \omega_{BC}^2 (r\cos 40^\circ \hat{i} + r\sin 40^\circ \hat{j})$$

$$= a_{BX}\hat{i} + (\alpha_{BC}\, r)(\cos 40^\circ \hat{j} - \sin 40^\circ \hat{i}) - (\omega_{BC}^2\, r)(\cos 40^\circ \hat{i} + \sin 40^\circ \hat{j})$$

(5) $\hat{i}$) $a_{BCX} = a_{BX} - \alpha_{BC}\, r \sin 40^\circ - \omega_{BC}^2\, r \cos 40^\circ$

(6) $\hat{j}$) $a_{BCy} = +\alpha_{BC}\, r \cos 40^\circ - \omega_{BC}^2\, r \sin 40^\circ$

6 Unknowns are: $a_{BCX}, a_{BCy}, M_B, B_X, B_y, a_{BX}$

Gets moment $\vec{M}_B = 17.2$ N-m $\hat{k}$

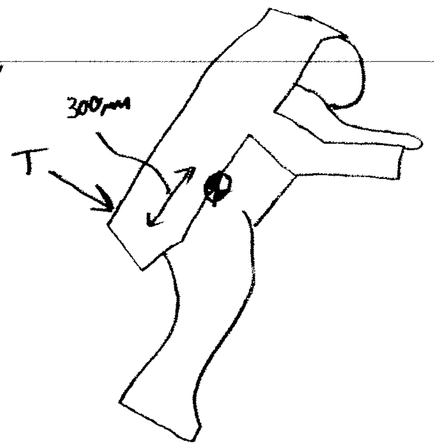
reaction $\vec{B} = -35.4\,N\,\hat{i} + 97.4\,N\,\hat{j}$

7.11 Astronaut

Thruster fires for 1 sec @ $T = 14.2N$

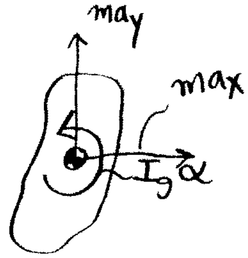
1 rev takes 60 sec

Find I_G



FBD

KD



CAM rate (sys = all, pt = com, ↺+) $\frac{d}{dt}(\vec{L}_{sys,com}) = \sum \vec{M}_{com}$

① $I_g \alpha = (T)(0.3m)$ unk: I_g, α

Kinematics: during the first second, α = const   using $\omega = \omega_0 + \alpha t$, $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$$\omega = \omega_0 \overset{0}{\nearrow} + \alpha t$$

② $\omega_1 = \alpha t_1$ unk: ω_1

integrate

③ $\theta_1 = \frac{1}{2}\alpha t_1^2$ unk: θ_1, α

next 59 seconds, $\alpha = 0$

④ $\omega_2 = \omega_1$ unk: ω_2

integrate

⑤ $\theta_2 = \omega_2 t_2$ unk: θ_2

together

⑥ $2\pi \text{ rad} = \theta_1 + \theta_2$ unk: $\theta_1 + \theta_2$

Solve

$$2\pi \text{ rad} = \frac{1}{2}\alpha t_1^2 + \omega_1 t_2$$
$$= \frac{1}{2}\alpha t_1^2 + (\alpha t_1) t_2$$
$$= \left(\frac{1}{2}\right)\alpha (1sec)^2 + (\alpha)(1sec)(59sec)$$
$$= \alpha\left(\frac{1}{2} + 59\right)$$

$$\alpha = 0.1056 \text{ rad/s}^2$$

into ① to get $I_g = 40.34 \text{ kgm}^2$