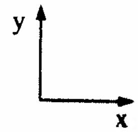
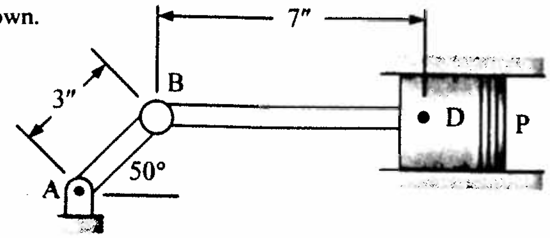
Example 4/12 version 2

Bar AB rotates with [redacted] an angular velocity of 1600 rev/min counterclockwise, and angular accel of 65 rev/s^2 CCW. Determine the [redacted] accel of point D at the instant shown.

From before, we found
$\omega_{BD} = 46.16 \text{ rad/sec}$ clockwise





To find $\vec{a}_D$, we need $\vec{a}$ of another point on bar BD → pt B

Kinematics of AB

$$\vec{a}_B = \underbrace{\vec{a}_A}_{0} + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$

$$\vec{\alpha}_{AB} = 65 \frac{rev}{s^2}\hat{k}$$

$$\vec{r}_{B/A} = \left(\tfrac{3}{12} ft\right)(\cos 50° \hat{\imath} + \sin 50° \hat{\jmath}) = 0.1607 ft\,\hat{\imath} + 0.1915 ft\,\hat{\jmath}$$

$$\omega_{AB} = \frac{1600 rev}{min} \frac{1 min}{60 sec} \frac{2\pi rad}{1 rev} = 167.6 \text{ rad/s}$$

$$\vec{\alpha}_{AB} = \frac{65 rev}{s^2} \frac{2\pi rad}{1 rev} = 408.4 \text{ rad/s}^2 \hat{k}$$

$$\vec{a}_B = \left(408.4 \tfrac{rad}{s^2}\hat{k}\right) \times (0.1607 ft\,\hat{\imath} + 0.1915 ft\,\hat{\jmath}) - \left(167.6 \tfrac{rad}{s}\right)^2 (0.1607 ft\,\hat{\imath} + 0.1915 ft\,\hat{\jmath})$$

$$= (-78.2 ft/s^2 - 4514 ft/s^2)\hat{\imath} + (65.63 ft/s^2 - 5351 ft/s^2)\hat{\jmath}$$

$$\vec{a}_B = -4592.2 ft/s^2 \hat{\imath} - 5285.4 ft/s^2 \hat{\jmath}$$

Kinematics of bar BD

$$\vec{a}_D = \vec{a}_B + \vec{\alpha}_{BD} \times \vec{r}_{D/B} - \omega_{BD}^2 \vec{r}_{D/B}$$

$$\vec{r}_{D/B} = \tfrac{7}{12} ft\,\hat{\imath} \qquad \vec{a}_D = a_D \hat{\imath}$$

$$\omega_{BD} = 46.16 \text{ rad/s}$$

$$\vec{\alpha}_{BD} = \alpha_{BD}\hat{k}$$

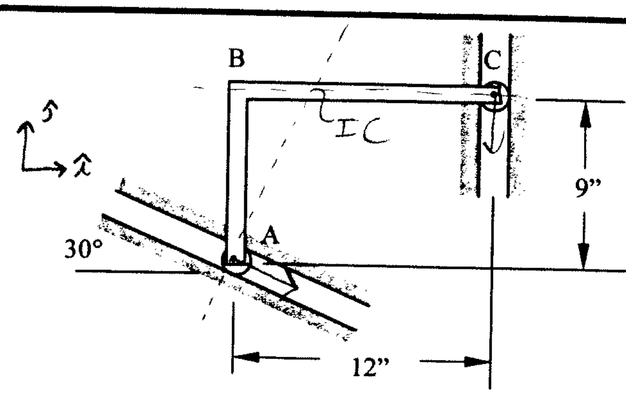
$$\vec{\alpha}_{BD} \times \vec{r}_{D/B} = (\alpha_{BD}\hat{k}) \times \left(\tfrac{7}{12} ft\,\hat{\imath}\right) = \left(\tfrac{7}{12}\alpha\, ft\right)\hat{\jmath}$$

$\hat{\imath}$) $a_D = -4592.2 ft/s^2 - \left(46.16 \tfrac{rad}{s}\right)^2 \left(\tfrac{7}{12} ft\right)$

$$\boxed{\vec{a}_D = -5835 ft/s^2 \hat{\imath}}$$

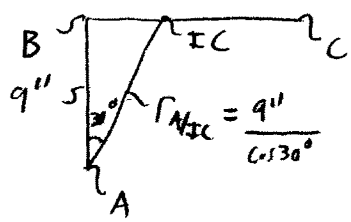
Example 4/19

The velocity of point A is 10 in/s down the slot and its acceleration is 100 in/s^2 up the slot when in the position shown. Determine the acceleration of point C.



The accel relationship involves ω, so lets use kinematics to find it

Kinematics of ABC, using IC



$V_A = \omega r_{A/IC}$

$10 \text{ in/s} = \omega (r_{A/IC})$;, $\omega = \frac{10 \text{ in/s}}{r_{A/IC}}$

$\omega = \frac{(10 \text{ in/s})}{(9 \text{in}/\cos 30°)} = 0.962 \text{ rad/s}$

Kinematics of accel

$$\vec{a}_C = \vec{a}_A + \vec{\alpha} \times \vec{r}_{C/A} - \omega^2 \vec{r}_{C/A}$$

$\vec{a}_A = (100 \text{ in/s}^2)(-\cos 30° \hat{i} + \sin 30° \hat{j}) = -86.6 \text{ in/s}^2 \hat{i} + 50 \text{ in/s}^2 \hat{j}$

$\vec{r}_{C/A} = 12 \text{in} \hat{i} + 9 \text{in} \hat{j}$

$\vec{\alpha} = \alpha \hat{k}$

$\vec{a}_C = a_C \hat{j}$

$$a_C \hat{j} = -86.6 \text{ in/s}^2 \hat{i} + 50 \text{ in/s}^2 \hat{j} + (\alpha \hat{k}) \times (12 \text{in} \hat{i} + 9 \text{in} \hat{j}) - \left(0.962 \tfrac{\text{rad}}{\text{s}}\right)^2 (12 \text{in} \hat{i} + 9 \text{in} \hat{j})$$

$$'' \quad + (12 \text{in}\, \alpha) \hat{j} - (9 \text{in}\, \alpha) \hat{i} - 11.1 \text{ in/s}^2 \hat{i} - 8.33 \text{ in/s}^2 \hat{j}$$

$\hat{i}$) $0 = -86.6 \text{ in/s}^2 - 9 \text{in}\, \alpha - 11.1 \text{ in/s}^2 \Rightarrow \alpha = -10.86 \text{ rad/s}^2$

$\hat{j}$) $a_C = 50 \text{ in/s}^2 + 12 \text{in}\, \alpha - 8.33 \text{ in/s}^2$

$= 50 + (12)(-10.86) - 8.33$

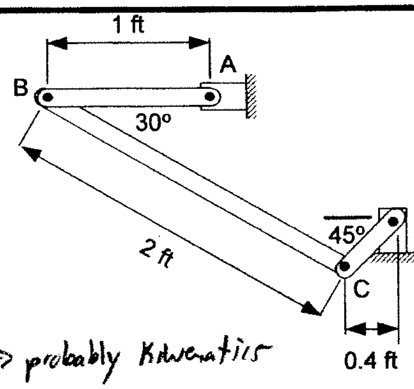
$= -88.6 \text{ in/s}^2$

$$\boxed{\vec{a}_C = -88.6 \text{ in/s}^2 \hat{j}}$$

vector approach

Example 4/20

Bar CD rotates with a constant angular velocity of 10 rad/s counterclockwise. Determine the angular velocity and angular acceleration of bars AB and BC at the instant shown.



Strategy: No masses or forces, want accel + vel ⇒ probably Kinematics

Find: $\vec{\omega}_{AB}$, $\vec{\omega}_{BC}$, $\vec{\alpha}_{AB}$, $\vec{\alpha}_{BC}$

Eqn	Unk
1	a_{BX}, ω_{AB}
2	a_{BY}, α_{AB}
3	v_{BX}
4	v_{BY}
5	$a_{CX}, \alpha_{BC}, \omega_{BC}$
6	a_{CY}
7	v_{CX}
8	v_{CY}
9	
10	
11	
12	

Soln: Kinematics of AB

$$\vec{a}_B = \underbrace{\vec{a}_A}_{0} + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{\alpha}_{AB} \times (-1\text{ft}\,\hat{\imath}) - \omega_{AB}^2 (-1\text{ft}\,\hat{\imath})$$

using $\vec{\alpha}_{AB} = \alpha_{AB} \hat{k}$

$$\vec{a}_B = (-\alpha_{AB})(1\text{ft})\hat{\jmath} + (1\text{ft})\,\omega_{AB}^2\, \hat{\imath}$$

$\hat{\imath}$) $a_{BX} = (1\text{ft})\,\omega_{AB}^2$ ①

$\hat{\jmath}$) $a_{BY} = (1\text{ft})(-\alpha_{AB})$ ②

consider velocity relation

$$\vec{v}_B = \underbrace{\vec{v}_A}_{0} + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$v_{BX}\hat{\imath} + v_{BY}\hat{\jmath} = (\omega_{AB}\hat{k}) \times (-1\text{ft}\,\hat{\imath})$$
$$= (-1\text{ft})\,\omega_{AB}\,\hat{\jmath}$$

$\hat{\imath}$) $v_{BX} = 0$ ③

$\hat{\jmath}$) $v_{BY} = (-1\text{ft})\,\omega_{AB}$ ④

Kinematics of BC

$$\vec{a}_B = \vec{a}_C + \vec{\alpha}_{BC} \times \vec{r}_{B/C} - \omega_{BC}^2 \vec{r}_{B/C}$$

using $\vec{r}_{B/C} = (2\text{ft})(-\cos 30^\circ\hat{\imath} + \sin 30^\circ\hat{\jmath}) = -1.732\text{ft}\,\hat{\imath} + 1\text{ft}\,\hat{\jmath}$

$$\vec{a}_B = \vec{a}_C + (\alpha_{BC}\hat{k}) \times (-1.732\text{ft}\,\hat{\imath} + 1\text{ft}\,\hat{\jmath}) - \omega_{BC}^2(-1.732\text{ft}\,\hat{\imath} + 1\text{ft}\,\hat{\jmath})$$

$$\vec{a}_B = \vec{a}_C + (-1.732\text{ft}\,\alpha_{BC})\hat{\jmath} + (-1\text{ft}\,\alpha_{BC}\,\hat{\imath}) + \omega_{BC}^2(1.732\text{ft}\,\hat{\imath} - 1\text{ft}\,\hat{\jmath})$$

$\hat{\imath}$) $a_{BX} = a_{CX} - (1\text{ft})(\alpha_{BC}) + \omega_{BC}^2(1.732\text{ft})$ ⑤

$\hat{\jmath}$) $a_{BY} = a_{CY} - 1.732\text{ft}\,\alpha_{BC} - \omega_{BC}^2(1\text{ft})$ ⑥

consider velocity relation

$$\vec{v}_B = \vec{v}_C + \vec{\omega}_{BC} \times \vec{r}_{B/C}$$

$$\vec{v}_B = \vec{v}_C + (\omega_{BC}\hat{k}) \times (-1.732\,ft\,\hat{\imath} + 1\,ft\,\hat{\jmath})$$

$\hat{\imath}$) $v_{Bx} = v_{Cx} - (1ft)\omega_{BC}$ ⑦

$\hat{\jmath}$) $v_{By} = v_{Cy} - (1.732ft)\omega_{BC}$ ⑧

Kinematics of CD

$$\vec{a}_C = \underset{\to 0}{\vec{a}_D} + \underset{\to 0}{\vec{\alpha}_{CD}} \times \vec{r}_{C/D} - \omega_{CD}^2 \vec{r}_{C/D}$$

$$\vec{a}_C = -\omega_{CD}^2(-.4ft\,\hat{\imath} - .4ft\,\hat{\jmath})$$

$\hat{\imath}$) $a_{Cx} = (.4ft)(10\,rad/s)^2 = 40\,ft/s^2$ ⑨

$\hat{\jmath}$) $a_{Cy} = (.4ft)(10\,rad/s)^2 = 40\,ft/s^2$ ⑩

consider velocity relation

$$\vec{v}_C = \underset{\to 0}{\vec{v}_D} + \vec{\omega}_{CD} \times \vec{r}_{C/D}$$

$$\vec{v}_C = (10\,rad/s\,\hat{k}) \times (-.4ft\,\hat{\imath} - .4ft\,\hat{\jmath})$$

$\hat{\imath}$) $v_{Cx} = 4\,ft/s\,\hat{\imath}$ ⑪

$\hat{\jmath}$) $v_{Cy} = -4\,ft/s\,\hat{\jmath}$ ⑫

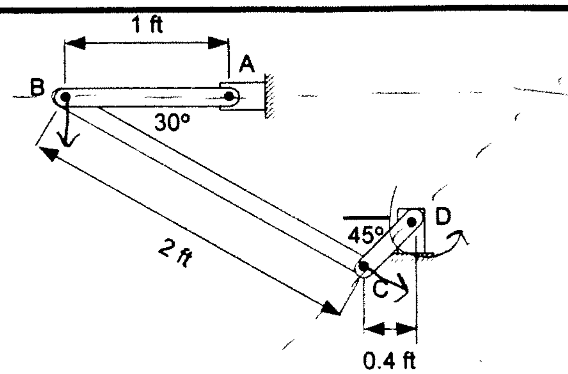
Count up 12 eqns and 12 unknowns, solve in MAPLE

$$\boxed{\begin{aligned}\vec{\alpha}_{AB} &= -113.6\,rad/s^2\,\hat{k}\\ \vec{\alpha}_{BC} &= -51.7\,rad/s^2\,\hat{k}\end{aligned}}$$

IC approach

Example 4/20

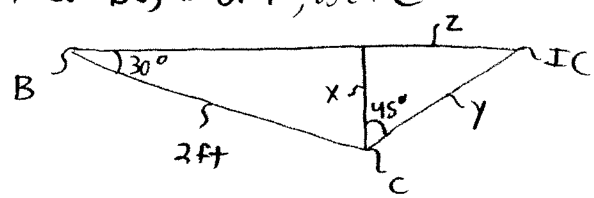
Bar CD rotates with a constant angular velocity of 10 rad/s counterclockwise. Determine the angular velocity and angular acceleration of bars AB and BC at the instant shown.



Strategy: No masses or forces given, asked for velocity & accel ⇒ Kinematics

Sol: Kinematics of Bar CD, in FAR $\quad V_C = \omega_{CD} r_{C/D} = \left(10 \frac{rad}{s}\right)\left(\frac{0.4 ft}{\cos 45°}\right) = 5.657 \text{ ft/s}$

Kinematics of Bar BC, in GPM, use IC



$x = 2ft \sin 30° = 1 ft$

$y = x/\cos 45° = 1.414 \text{ ft}$ $\quad$ this is $r_{C/IC}$

$z = y \sin 45° = 1 ft$

$V_C = \omega_{BC} r_{C/IC} \Rightarrow 5.657 \text{ ft/s} = (\omega_{BC})(1.414 ft) \quad \therefore \omega_{BC} = 4 \text{ rad/sec}$

$V_B = \omega_{BC} r_{B/IC} = \omega_{BC}(z + 2ft \cos 30°) = (4 \text{ rad/s})(2.732 ft) = 10.93 \text{ ft/s}$

Kinematics of Bar AB, in FAR

$V_B = \omega_{AB} r_{B/A} \Rightarrow 10.93 \text{ ft/s} = \omega_{AB}(1ft) \Rightarrow \omega_{AB} = 10.93 \text{ rad/s}$

Kinematics of Bar CD, in FAR

$\vec{a}_C = \vec{a}_D + \vec{\alpha}_{CD} \times \vec{r}_{C/D} - \omega_{CD}^2 \vec{r}_{C/D} = -\left(10 \text{ rad/s}\right)^2(-.4ft\,\hat{i} - .4ft\,\hat{j})$

($\vec{a}_D \to 0$, $\vec{\alpha}_{CD} \to 0$)

$\vec{a}_C = 40 \text{ ft/s}^2\,\hat{i} + 40 \text{ ft/s}^2\,\hat{j}$

Kinematics of Bar BC, in GPM

$\vec{a}_B = \vec{a}_C + \vec{\alpha}_{BC} \times \vec{r}_{B/C} - \omega_{BC}^2 \vec{r}_{B/C}$

$\vec{\alpha}_{BC} = \alpha_{BC} \hat{k}$

$\vec{r}_{B/C} = 2ft\left(-\cos 30°\,\hat{i} + \sin 30°\,\hat{j}\right) = -1.732 ft\,\hat{i} + 1ft\,\hat{j}$

$$a_{Bx}\hat{\imath} + a_{By}\hat{\jmath} = 40\,\text{ft/s}^2\,\hat{\imath} + 40\,\text{ft/s}^2\,\hat{\jmath} + (\alpha_{BC}\hat{k})\times(-1.732\,\text{ft}\,\hat{\imath} + 1\,\text{ft}\,\hat{\jmath})$$
$$- (4\,\text{rad/s})^2(-1.732\,\text{ft}\,\hat{\imath} + 1\,\text{ft}\,\hat{\jmath})$$

$$'' = '' + (-1.732\,\text{ft}\,\alpha_{BC}\hat{\jmath}) - (\alpha_{BC}\,\text{ft}\,\hat{\imath}) + 27.712\,\text{ft/s}^2\,\hat{\imath} - 16\,\text{ft/s}^2\,\hat{\jmath}$$

$\hat{\imath}$) $a_{BX} = 40\,\text{ft/s}^2 - (1\,\text{ft})\alpha_{BC} + 27.712\,\text{ft/s}^2$ ①

$\hat{\jmath}$) $a_{BY} = 40\,\text{ft/s}^2 - 1.732\,\text{ft}\,\alpha_{BC} - 16\,\text{ft/s}^2$ ②

Kinematics of Bar AB, FAR

$$\vec{a}_B = \underbrace{\vec{a}_A}_{\to 0} + \vec{\alpha}_{AB}\times\vec{r}_{B/A} - \omega_{AB}^2\,\vec{r}_{B/A}$$

$$\vec{\alpha}_{AB} = \alpha_{AB}\hat{k}$$

$$\vec{r}_{B/A} = -1\,\text{ft}\,\hat{\imath}$$

$$a_{Bx}\hat{\imath} + a_{By}\hat{\jmath} = (\alpha_{AB}\hat{k})\times(-1\,\text{ft}\,\hat{\imath}) - (10.93\,\text{rad/s})^2(-1\,\text{ft}\,\hat{\imath})$$
$$= (-1\,\text{ft})\,\alpha_{AB}\hat{\jmath} + 119.5\,\text{ft/s}^2\,\hat{\imath}$$

$\hat{\imath}$) $a_{BX} = 119.5\,\text{ft/s}^2$ ③

$\hat{\jmath}$) $a_{By} = (-1\,\text{ft})\,\alpha_{AB}$ ④

4 eqns, unknowns are $a_{BX}, a_{BY}, \alpha_{BC}, \alpha_{AB}$ solving gets

$$\boxed{\vec{\alpha}_{BC} = -51.7\,\text{rad/s}^2\,\hat{k} \qquad \vec{\alpha}_{AB} = -113.6\,\text{rad/s}^2\,\hat{k}}$$

Big Picture:

- No masses or forces, asked for vel + accel ⇒ Kinematics
- for each body there is an accel + vel relation
- use the shared points between bodies to link the equations
- every vector equation holds two independent equations inside