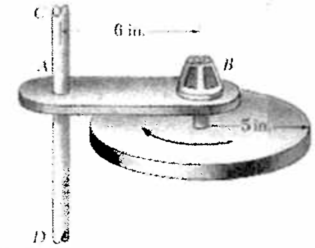
17.95 A 10-lb disk is attached to the shaft of a motor mounted on arm AB which is free to rotate about the vertical axle CD. The arm-and-motor unit has a moment of inertia of 0.032 lb ft s^2 with respect to the axle CD, and the normal operating speed of the motor is 360 rpm. Knowing that the system is initially at rest, determine the angular velocities of the arm and of the disk when the motor reaches a speed of 360 rpm.
(*taken from Vector Mechanics for Engineers, 5th Edition by Beer & Johnston*)



Strategy: Kinematics
$CAM_{F.T.}$

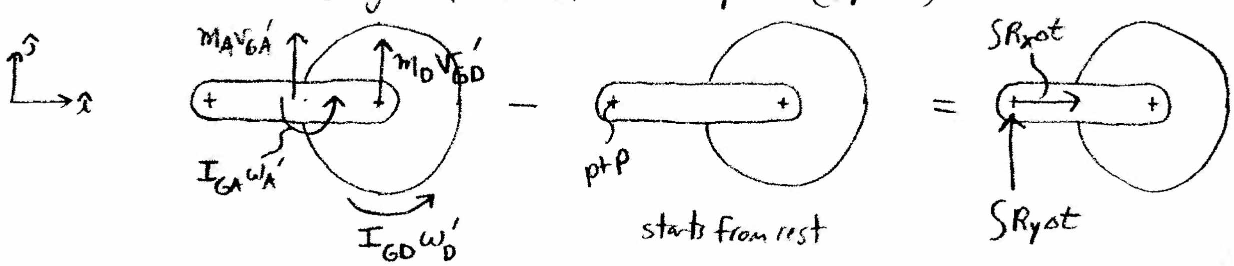
Solution: First, some background calculations

Denote the arm + motor as A, disk as D

$$\omega_M = \left(\frac{360\,rev}{min}\right)\left(\frac{1\,min}{60\,sec}\right)\left(\frac{2\pi\,rad}{1\,rev}\right) = 12\pi\,\frac{rad}{sec}$$ motor angular speed

$$I_{GD} = \frac{1}{2} m_D r^2$$
$$= \left(\frac{1}{2}\right)\left(10\,lb/32.2\,ft/s^2\right)\left(5\,in/12\,in/ft\right)^2 = 0.027\ lb\text{-}ft\text{-}s^2$$

Momentum Diagram from rest to full speed (top view)



Both the arm A and disk D will rotate
Everything starts from rest

$CAM_{F.T.}$ (sys = arm + motor + disk, about pt P on axis CD, (+↺))

$$L_{sys,2P} - L_{sys\,1,P} = \Sigma M_P$$

$$\left[I_{GA}\omega_A' + I_{GD}\omega_D' + m_D v_{GD}'(6\,in) + m_A v_{GA}'(r_{GA/P})\right] - [0] = [0]$$

Note: $\omega_D' \neq \omega_M$, $\omega_D' = \omega_{disk}'$
$\omega_M' = \omega_{motor} = \omega_{disk/arm}$

in analogy to relative velocity $\omega_{disk/arm} = \omega_{disk} - \omega_{arm}$

$$\omega_M = \omega_D' - \omega_A' \quad \text{so} \quad \omega_D' = \omega_M + \omega_A'$$

Also, $r_{GA/P}$ is distance from point P to COM of bar AB + motor.

Masses of bar AB and motor are not given, so just leave it an unknown.

Later we will realize $I_o = I_{GA} + m_A r_{GA/P}^2$, given as 0.032 lb-ft-s^2

CAM eqn is now

$$① \quad I_{GA}\omega_A' + I_{GD}(\omega_M + \omega_A') + m_D V_{GD}'\left(\tfrac{6}{12}\text{ft}\right) + m_A V_{GA}'(r_{GA/P}) = 0$$

Now consider kinematics of bar AB (fixed axis rotation)

$$② \quad V_{GA}' = (\omega_A')(r_{GA/P}) = (r_{GA/P})\,\omega_A'$$

$$③ \quad V_{GD}' = (\omega_A')(\ell) = \left(\tfrac{6}{12}\text{ft}\right)\omega_A'$$

system is found here, the rest is solving for unknowns. You don't *need* Maple!

Plug ② + ③ into ① to get

$$I_{GA}\omega_A' + I_{GD}(\omega_M + \omega_A') + m_D\left(\tfrac{6}{12}\text{ft}\right)(\omega_A')\left(\tfrac{6}{12}\text{ft}\right) + m_A(r_{GA/P}\,\omega_A')(r_{GA/P}) = 0$$

$$\omega_A'\left[I_{GA} + I_{GD} + m_D\left(\tfrac{1}{2}\text{ft}\right)^2 + m_A(r_{GA/P})^2\right] = -I_{GD}\,\omega_M$$

$$\omega_A'\left[\underbrace{I_{GA} + m_A r_{GA/P}^2}_{} + I_{GD} + m_D\left(\tfrac{1}{2}\text{ft}\right)^2\right] = -I_{GD}\,\omega_M$$

this is the mass moment of inertia of the arm AB + motor with respect to the axle CD, given as $0.032 \text{ lb-ft-s}^2 = I_o$

$$\omega_A'\left[I_o + I_{GD} + m_D\left(\tfrac{1}{2}\text{ft}\right)^2\right] = -I_{GD}\,\omega_M$$

$$\omega_A' = \frac{-I_{GD}\,\omega_M}{I_o + I_{GD} + m_D\left(\tfrac{1}{2}\text{ft}\right)^2}$$

$$= \frac{(-0.027 \text{ lbfts}^2)\left(12\pi \tfrac{\text{rad}}{\text{s}}\right)}{(0.032 \text{ lbfts}^2) + (0.027 \text{ lbfts}^2) + \left(\tfrac{10}{32.2}\text{slugs}\right)\left(\tfrac{1}{2}\text{ft}\right)^2}$$

$$= \frac{-1.01788 \text{ lbfts}^2\left(\tfrac{\text{rad}}{\text{s}}\right)}{0.13664 \text{ lbfts}^2} = -7.45 \text{ rad/s}$$

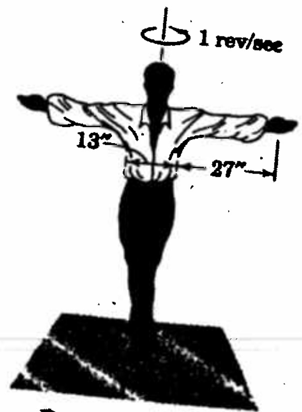
$$\boxed{\vec{\omega}_{Arm} = -7.45 \text{ rad/s } \hat{k}}$$

$$\omega_{disk} = \omega_M + \omega_{Arm} = \left(12\pi \tfrac{\text{rad}}{\text{s}}\right) + \left(-7.45 \tfrac{\text{rad}}{\text{s}}\right) = 30.25 \tfrac{\text{rad}}{\text{s}}$$

$$\boxed{\vec{\omega}_{disk} = 30.25 \tfrac{\text{rad}}{\text{s}} \hat{k}}$$

So disk + arm spin in opposite directions

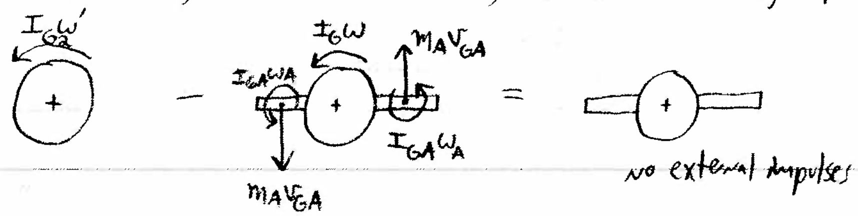
6/188 The 165-lb ice skater with arms extended horizontally spins about a vertical axis with a rotational speed of 1 rev/sec. Estimate his rotational speed N if he fully retracts his arms, bringing his hands very close to the centerline of his body. As a reasonable approximation, model the extended arms as uniform slender rods, each of which is 27 in. long and weighs 15 lb. Model the torso as a solid 135-lb cylinder 13 in. in diameter. Treat the man with arms retracted as a solid 165-lb cylinder of 13-in. diameter. Neglect friction at the skate-ice interface.



Problem 6/188

Strategy: $CAM_{\text{finite time}}$

Solution: system = skater, ① arms outstretched, ② arms retracted, topview



$$I_{G2} = \tfrac{1}{2} m_2 r_2^2 = \left(\tfrac{1}{2}\right)\left(165/32.2 \text{ slugs}\right)\left(\tfrac{13}{2\cdot 12} \text{ ft}\right)^2 = 0.7517 \text{ slugs ft}^2$$

$$I_G = \tfrac{1}{2} m r^2 = \left(\tfrac{1}{2}\right)\left(135/32.2 \text{ slugs}\right)\left(\tfrac{13}{2\cdot 12} \text{ ft}\right)^2 = 0.6151 \text{ slugs ft}^2$$

$$I_{GA} = \tfrac{1}{12} m_A L^2 = \left(\tfrac{1}{12}\right)\left(\tfrac{15}{32.2} \text{ slugs}\right)\left(\tfrac{27}{12} \text{ ft}\right)^2 = 0.1965 \text{ slugs ft}^2$$

$CAM_{F.T.}$ about com of skater, (+)

$$L_{sys,2} - L_{sys,1} = \underbrace{\Sigma \int M\,dt}_{0}$$

$$\left[I_{G2}\omega'\right] - \left[2\left(I_{GA}\omega_A + m_A v_{GA}\left(\tfrac{6.5+13.5}{12} \text{ ft}\right)\right) + I_G\omega\right] = 0$$

① $$I_{G2}\omega' = I_G\omega + 2\left[I_{GA}\omega_A + m_A v_{GA}\left(\tfrac{5}{3} \text{ ft}\right)\right]$$

Given $\omega = \left(\frac{1 \text{ rev}}{\text{sec}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 2\pi \text{ rad/sec}$

Kinematics

$$v_{GA} = \omega\left(\tfrac{D}{2} + \tfrac{L}{2}\right) = \left(2\pi \tfrac{\text{rad}}{\text{sec}}\right)\left(\tfrac{5}{3} \text{ ft}\right) = \tfrac{10\pi}{3} \text{ ft/s}$$

plug $\omega = 2\pi$ rad/sec and $V_{GA} = \frac{10\pi}{3}$ ft/s into Eqn ①

$$I_{G2}\,\omega' = I_G\left(2\pi \tfrac{rad}{sec}\right) + 2\left[I_{GA}\left(2\pi \tfrac{rad}{sec}\right) + m_A\left(\tfrac{10\pi}{3}\tfrac{ft}{s}\right)\left(\tfrac{5}{3}ft\right)\right]$$

$$(0.7517)\,\omega' = (0.6151)(2\pi) + 2\left[(0.1965)(2\pi) + \left(\tfrac{15}{32.2}\right)\left(\tfrac{10\pi}{3}\right)\left(\tfrac{5}{3}\right)\right]$$

$$\omega' = 30.1 \tfrac{rad}{s} \quad \text{or} \quad \boxed{\vec{\omega}' = 30.1 \tfrac{rad}{s}\,\hat{K}}$$

If you analyze this change using Conservation of Energy, you will see that $KE_1 \neq KE_2$. The difference comes from a change in internal energy of the skater, because $U_1 \neq U_2$. Considered together, you will get $\Delta KE = \Delta U$