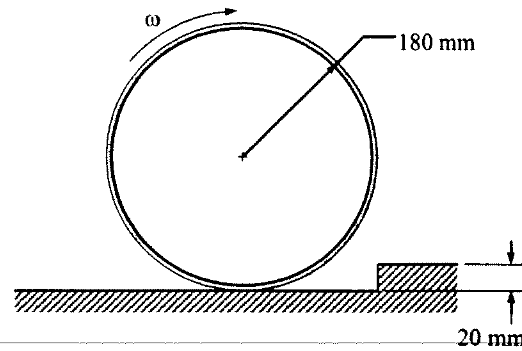
Example 4/16

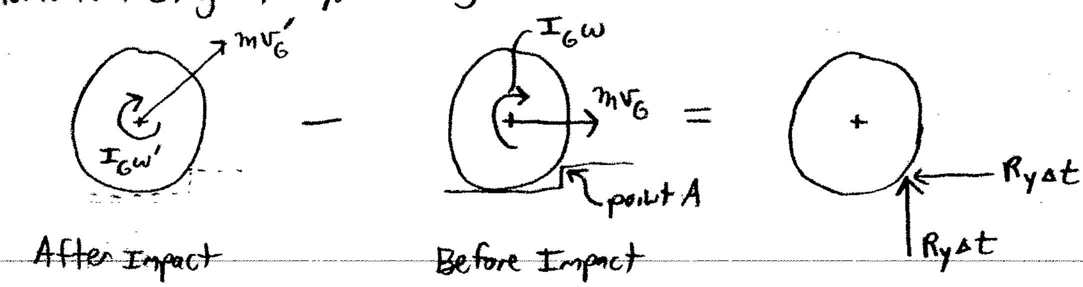
A thin ring having a mass of 15 kg strikes the 20-mm high step. The mass moment of inertia of a ring is mr^2. Determine the smallest angular velocity ω the ring can have so that it will make it over the step. Assume the ring does not slip or rebound from the step.



Strategy: There is an impact with the step, so use $CLM_{F.T.}$ and $CAM_{F.T.}$

Solution: $I_G = mr^2 = (15\,kg)(.18\,m)^2 = 0.486\ kg\,m^2$

Momentum Diagram system = ring



the disk is in fixed axis rotation after the impact, so the velocity of the COM is perpendicular to a line drawn from the pivot point

We are not asked for the impulsive reaction forces due to the step, so let's choose the pivot point (call it pt. A) to take moments about.

$CAM_{F.T.}$ (sys = ring, pt A, ↻)

$$L_{sys,A2} - L_{sys,A1} = \Sigma M_A \Delta t$$

$$\left[I_G\omega' + (mv_G')(r)\right] - \left[I_G\omega + (mv_G)(r - 0.02\,m)\right] = 0 \qquad (1)$$

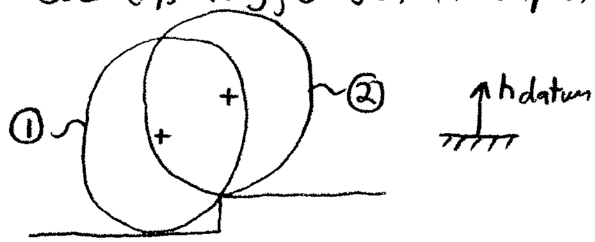
Use kinematics to relate v_G + ω and v_G' + ω', IC = contact pt

$$v_G = \omega r \qquad (2)$$

$$v_G' = \omega' r \qquad (3)$$

For the ring to clear the step, it must have enough energy → use CoE

CoE (sys = ring, ① = just after impact, ② = when step is cleared)



Only external forces are reactions at pt A, but they do no work
∴ $W_{ext} = 0$

$$E_{sys,1} = KE_1 + \underbrace{\cancel{PE_1}_{\to 0} + \cancel{SE_1}_{\to 0} + \cancel{U_1}_{\to 0}}_{\text{by our datum definition}}$$

$$KE_1 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

$$E_{sys,2} = KE_2 + PE_2 + \cancel{SE_2}_{\to 0} + \cancel{U_1}_{\to 0}$$

$$PE_2 = mg\,(0.02\,m)$$

$KE_2 = 0$ because smallest ω so that the ring makes it over the step will have it stop at the top of the step

$$E_{sys,2} - E_{sys,1} = W_{ext}$$

$$mg\,(0.02\,m) - \left[\frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2\right] = 0 \qquad (4)$$

4 equations and 4 unknowns, can solve to get

$$\boxed{\vec{\omega} = 2.61 \ \text{rad/s} \ (-\hat{k})}$$

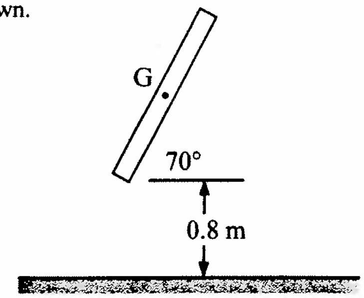
Example 4/17

A slender bar 1.5 meters long and weighing 10 kg is dropped onto a horizontal surface as shown. Assume the friction between the bar and the ground is large. Immediately after the impact:

a) Determine the angular velocity of the bar for e = 0.5

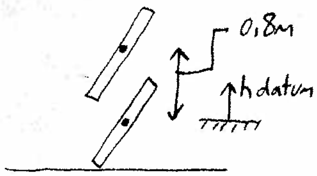
b) Plot the angular velocity as a function of the coefficient of restitution for e = 0 to e = 1.

b) How does your answer change if the surface is frictionless?



Approach: CLM_{FT} and/or CAM_{FT}, CoR across the impact
CoE during freefall

Solution: CoE from release ① to just before impact ②, system = bar, set datum to zero at position ②



$$E_{sys,1} = \underbrace{KE_1}_{\to 0} + PE_1 + \underbrace{SE_1}_{\to 0} + \underbrace{U_1}_{\to 0}$$

$$PE_1 = mg(0.8m)$$

$$E_{sys,2} = KE_2 + \underbrace{PE_2}_{\to 0} + \underbrace{SE_2}_{\to 0} + \underbrace{U_2}_{\to 0}$$

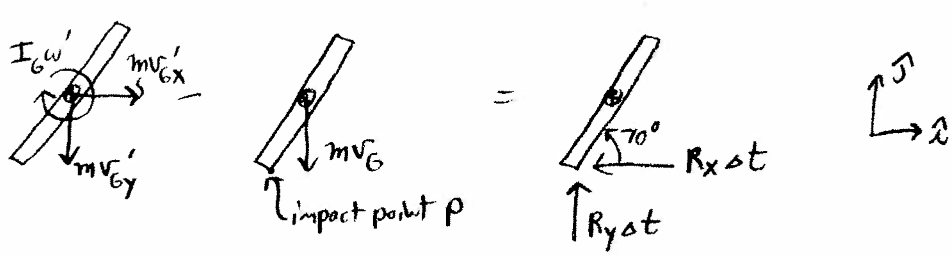
$$KE_2 = \tfrac{1}{2} m V_G^2 + \underbrace{\tfrac{1}{2} I_G \omega^2}_{\text{zero b/c } \omega = 0 \text{ before impact}}$$

$W_{ext} = 0$ (no external forces)

$$E_{sys,2} - E_{sys,1} = W_{ext}$$

$$\tfrac{1}{2} m v_G^2 - mg(0.8m) = 0 \quad \Rightarrow \quad V_G = \sqrt{2g(0.8m)}$$ velocity of COM just before impact

Draw the Impulse-Momentum Diagram

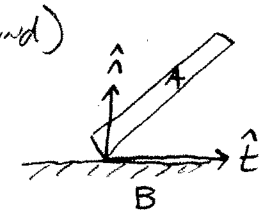


$CAM_{F.T.}$ (about pt P, sys = bar, ↻)

$$L_{sys,P} - L_{sys,P} = \Sigma M_P$$

$$\left[mV_{Gy}'\left(\tfrac{L}{2}\cos 70°\right) + mV_{Gx}'\left(\tfrac{L}{2}\sin 70°\right) + I_G\omega'\right] - \left[mV_G\left(\tfrac{L}{2}\cos 70°\right)\right] = 0 \quad ①$$

Coefficient of Restitution (body A = bar, body B = ground)



$$e\left(v_{APn} - \underbrace{v_{BPn}}_{\to 0}\right) = \underbrace{v_{BPn}'}_{\to 0} - v_{APn}'$$

$$e\, v_{APn} = -\, v_{APn}'$$

use $v_{APn} = -v_G$ (rigid body translation)

$$e\, v_G = v_{APn}' \qquad 2$$

Use Kinematics $\vec{v}_{AP}' = \vec{v}_G' + \vec{\omega}' \times \vec{r}_{A/G}$

$$\vec{r}_{A/G} = -\tfrac{L}{2}\cos 70^\circ\,\hat{\imath} - \tfrac{L}{2}\sin 70^\circ\,\hat{\jmath}$$

$$\vec{\omega}' = \omega'(-\hat{k})$$

$$\vec{v}_G' = v_{Gx}'\,\hat{\imath} - v_{Gy}'\,\hat{\jmath}$$

$$\vec{v}_{AP}' = v_{APn}'\,\hat{\jmath}$$ (large friction prevents x-dir. velocity)

$$v_{APn}'\,\hat{\jmath} = \left(v_{Gx}'\,\hat{\imath} - v_{Gy}'\,\hat{\jmath}\right) + \left(-\omega'\hat{k}\right)\times\left(-\tfrac{L}{2}\right)\left(\cos 70^\circ\,\hat{\imath} + \sin 70^\circ\,\hat{\jmath}\right)$$

$$v_{APn}'\,\hat{\jmath} = \left(v_{Gx}'\,\hat{\imath} - v_{Gy}'\,\hat{\jmath}\right) + \left(\omega'\tfrac{L}{2}\right)\left(\cos 70^\circ\,\hat{\jmath} - \sin 70^\circ\,\hat{\imath}\right)$$

$\hat{\imath}$) $\quad 0 = v_{Gx}' - \omega'\tfrac{L}{2}\sin 70^\circ \qquad 3$

$\hat{\jmath}$) $\quad v_{APn}' = -v_{Gy}' + \omega'\tfrac{L}{2}\cos 70^\circ \qquad 4$

Unknowns are v_{Gy}', v_{Gx}', ω', v_{APn}' with 4 unknowns, solveable

$$\boxed{\omega' = 2.03\ \text{rad/s}\ (-\hat{k})}$$

b) if solved for general e, we get

$$\omega' = (1.355)e + 1.355$$

c) if frictionless, → remove impulse $R_x \Delta t$

$$\Rightarrow \vec{v}_{AP}' = v_{APt}'\,\hat{\imath} + v_{APn}'\,\hat{\jmath}$$

x-component of velocity of pt A

⇒ 5^{th} equation from CLM_{FT} in $\hat{\imath}$ direction