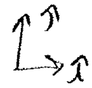
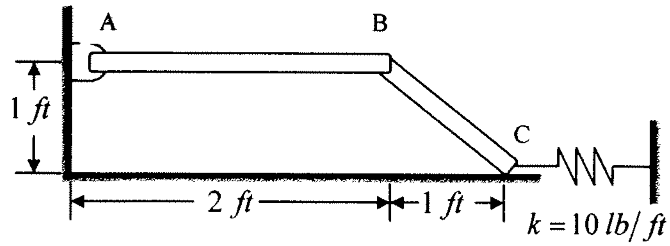
Ex. Bar AB weighs 10 lb and bar BC weighs 6 lb. If the system is released from rest in the position shown, what are the angular velocities of the bars at the instant just before joint B hits the smooth floor?





Known:

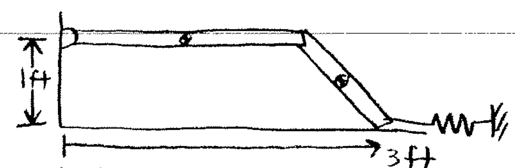
$m_{AB} = 0.311 \frac{lb\,s^2}{ft}$ $m_{BC} = 0.186 \frac{lb\,s^2}{ft}$ $I_{G,AB} = 0.104\,lb\,s^2\,ft$ $I_{G,BC} = 0.031\,lb\,s^2\,ft$

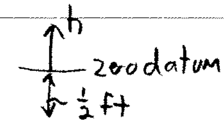
Approach: CoE from release ① to just before impact ② system = AB+BC+spring

$E_{sys,2} - E_{sys,1} = W_{ext}$

$W_{ext} = 0$ only forces are pin reactions (assume no friction)

$E_{sys,1} = KE_1 + PE_1 + \underbrace{SE_1}_{0 \text{ uncompressed}} + \underbrace{U_1}_{0 \text{ no impact}}$

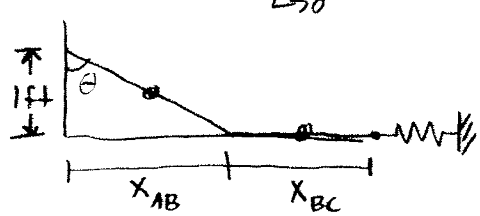


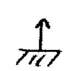


$PE_1 = m_{AB}\,g\left(\tfrac{1}{2}ft\right) + \underbrace{m_{BC}\,g(\emptyset)}_{\text{at datum}}$

$KE_1 = 0$ starting from rest

$E_{sys,2} = KE_2 + PE_2 + SE_2 + \underbrace{U_2}_{0}$





$X_{AB}^2 = 2^2 - 1^2 \quad \therefore X_{AB} = \sqrt{3ft^2} = 1.732\,ft$

$X_{BC}^2 = 1^2 + 1^2 \quad \therefore X_{BC} = \sqrt{2ft^2} = 1.414\,ft$

$X_{AB} + X_{BC} = 3.146\,ft \quad + \theta = 60°$

$SE_2 = \frac{1}{2}K\Delta x^2 = \left(\frac{1}{2}\right)\left(10\frac{lb}{ft}\right)\left(3.146ft - 3ft\right)^2 = 0.107\ lb\,ft$

$PE_2 = m_{BC}\,g\left(-\tfrac{1}{2}ft\right)$

$KE_2 = \tfrac{1}{2} m_{AB} v_{GAB}^2 + \tfrac{1}{2} m_{BC} v_{GBC}^2 + \tfrac{1}{2} I_{GAB}\,\omega_{AB}^2 + \tfrac{1}{2} I_{GBC}\,\omega_{BC}^2$

Together, CoE gives

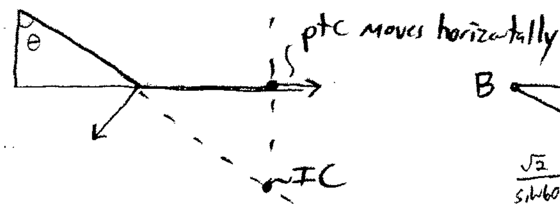
(1) $\tfrac{1}{2}\left[m_{AB} v_{GAB}^2 + m_{BC} v_{GBC}^2 + I_{GAB}\,\omega_{AB}^2 + I_{GBC}\,\omega_{BC}^2\right] - m_{BC}\,g\left(\tfrac{1}{2}ft\right) + 0.107\,lb\,ft - m_{AB}\,g\left(\tfrac{1}{2}ft\right) = 0$

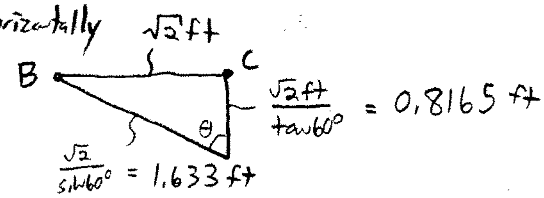
Kinematics of bar AB - fixed pivot at pt A

use $v = \omega r$ to get $v_{GAB} = \omega_{AB}\,(1ft)$ (2)

$v_B = \omega_{AB}\,(2ft)$ (3)

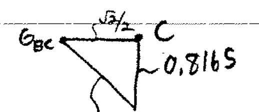
Kinematics of bar BC - GPM use IC





(4) $v_B = \omega_{BC}\,(1.633ft)$

(5) $v_{GBC} = \omega_{BC}\,(1.08ft)$



$r_{GBC/IC}^2 = \left(\frac{\sqrt{2}}{2}\right)^2 + (0.8165)^2$

$r_{GBC/IC} = 1.08\,ft$

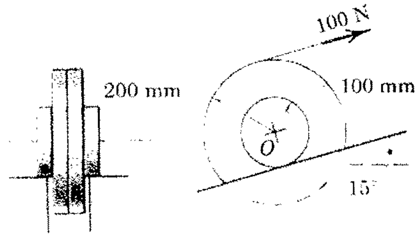
Unknowns: $v_{GAB}, v_{GBC}, \omega_{AB}, \omega_{BC}, v_B$

$\omega_{AB} = 4.479\,rad/s\;(-\hat{k})$

$\omega_{BC} = 5.487\,rad/s\;(\hat{k})$

Solving using vector approach (without IC) is much more complex

Ex. 6/9 The wheel rolls up the incline on its hubs without slipping and is pulled by the 100-N force applied to the cord wrapped around its outer rim. If the wheel start from rest, compute its angular velocity after its center has moved a distance of 3-m up the incline. The wheel has a mass of 40-kg with a center of mass at O and has a centroidal radius of gyration of 150-mm. (*taken from Engineering Mechanics, 4th Edition by Meriam & Kraige*)



$I_G = 0.9\ \text{kg m}^2$

Approach: CoE (~~two points in space~~) from start ① to 3m up slope ②

COE set PE datum to give zero PE at time ①, system = wheel only

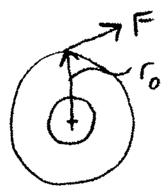
$$E_{sys,1} = \underbrace{KE_1}_{\to 0 \text{ starts from rest}} + \underbrace{PE_1}_{\to 0 \text{ by our defn of PE datum}} + \underbrace{SE_1}_{\to 0} + \underbrace{U_1}_{\to 0}$$

$$E_{sys,2} = KE_2 + PE_2 + \underbrace{SE_2}_{\to 0} + \underbrace{U_2}_{\to 0}$$

$PE_2 = mgh$ with $h = (3\text{m})\sin 15°$

$KE_2 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$

$$W_{ext} = \int_1^2 \vec{F} \cdot d\vec{s} = (\text{force})(\text{distance}) = (100\text{N})(9\text{m})$$



$$\text{distance} = (\#\text{revs})(\text{circumference at cord})$$
$$= \left(\frac{3\text{m}}{2\pi(0.1\text{m})}\right)(2\pi r_o)$$
$$= \left(\frac{30}{2\pi}\,[\text{rad}]\right)(2\pi(0.2\text{m})) = 6\text{m}$$

Together $E_{sys,2} - E_{sys,1} = W_{ext}$

$$mg(3\text{m} \sin 15°) + \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 - 0 = (100\text{N})(6\text{m}) \qquad ①$$

Kinematics

using contact point as IC and $\vec{v}_G = \vec{\omega} \times \vec{r}_{G/IC}$

$$\vec{v}_G = (-\omega \hat{k}) \times (0.1\text{m})(-\sin 15° \hat{\imath} + \cos 15° \hat{\jmath})$$

$$\left.\begin{aligned} v_{Gx} &= (\omega)(0.1\text{m})(\cos 15°) \\ v_{Gy} &= (\omega)(0.1\text{m})(\sin 15°) \end{aligned}\right\} v_G = (\omega)(0.1\text{m}) \qquad ②$$

gets $\vec{\omega} = 21.33$ rad/s $(-\hat{k})$

$v = 2.13$ m/s