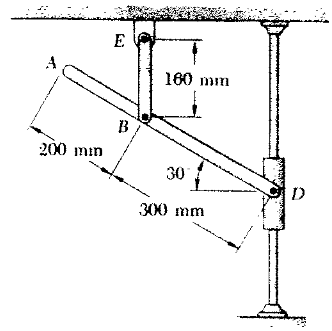


Fig. P15.59 and P15.60

Find: ω_{AD}
$\vec{V}_B$
$\vec{V}_A$

No masses or forces given, use Kinematics!

15.60 Knowing that at the instant shown the velocity of collar *D* is 1.8 m/s upward, determine (*a*) the angular velocity of rod *AD*, (*b*) the velocity of point *B*, (*c*) the velocity of point *A*.

Given $\vec{V}_D = 1.8\,\text{m/s}\ \hat{j}$

Constraint on pt B $\quad \vec{V}_B = V_B\,\hat{i}$ (rigid rod EB prevents vertical motion)

Relate $\vec{V}_D$ + $\vec{V}_B$: $\quad \vec{V}_B = \vec{V}_D + \vec{\omega}_{AD} \times \vec{r}_{B/D}$

$\vec{r}_{B/D} = (0.3\,\text{m})(-\cos 30°\,\hat{i} + \sin 30°\,\hat{j})$
$= -0.2598\,\hat{i} + 0.15\,\hat{j}\ \text{m}$

$\vec{\omega}_{AD} = \omega_{AD}\,\hat{k}$

$$V_B\,\hat{i} = 1.8\,\text{m/s}\,\hat{j} + (\omega_{AD}\,\hat{k}) \times (-0.2598\,\hat{i}\,\text{m} + 0.15\,\hat{j}\ \text{m})$$

$$V_B\,\hat{i} = 1.8\,\text{m/s}\,\hat{j} - 0.2598\,\text{m}\,\omega_{AD}\,\hat{j} - 0.15\,\text{m}\,\omega_{AD}\,\hat{i}$$

$\hat{i}$) $\quad V_B = (-0.15\,\text{m})\,\omega_{AD}$

$\hat{j}$) $\quad 0 = 1.8\,\text{m/s} - (0.2598\,\text{m})\,\omega_{AD}$

from $\hat{j}$ eqn so $\omega_{AD} = 6.928\ \text{rad/s}$ i.e. $\boxed{\vec{\omega}_{AD} = 6.928\,\hat{k}\ \frac{\text{rad}}{\text{s}}}$

Plug ω_{AD} into $\hat{i}$ eqn

$$V_B = (-0.15\,\text{m})(6.928\ \text{rad/s}) = -1.039\ \text{m/s}$$

$$\boxed{\vec{V}_B = -1.039\,\text{m/s}\ \hat{i}}$$

To get pt A: $\quad \vec{V}_A = \vec{V}_D + \vec{\omega}_{AD} \times \vec{r}_{A/D}$

$\vec{r}_{A/D} = (0.5\,\text{m})(-\cos 30°\,\hat{i} + \sin 30°\,\hat{j})$

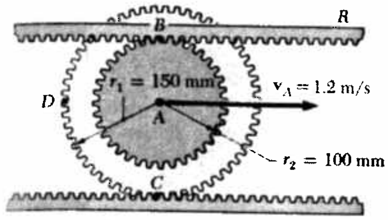
$$\vec{V}_A = (1.8\,\text{m/s})\,\hat{j} + (6.928\,\hat{k}\ \tfrac{\text{rad}}{\text{s}}) \times (0.5\,\text{m})(-\cos 30°\,\hat{i} + \sin 30°\,\hat{j})$$

$$= (1.8\,\text{m/s} - 3\,\text{m/s})\,\hat{j} + (-1.732\,\text{m/s})\,\hat{i}$$

$\boxed{\vec{V}_A = -1.732\,\text{m/s}\ \hat{i} - 1.2\,\text{m/s}\ \hat{j}}$ or $\vec{V}_A = 2.11\,\text{m/s}$ ↙ 34.7°

Group Work

Using IC idea

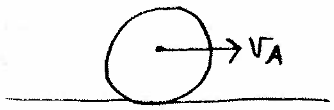


SAMPLE PROBLEM 15.2

The double gear shown rolls on the stationary lower rack; the velocity of its center A is 1.2 m/s directed to the right. Determine (*a*) the angular velocity of the gear, (*b*) the velocities of the upper rack R and of point D of the gear.

a) No masses, no forces ⇒ try kinematics $\quad \hat{\jmath}$ ↑, $\hat{\imath}$ →

Consider the rolling on stationary lower track



The gear teeth enforce no slipping, IC is contact pt

$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/IC}$ w/ $\vec{r}_{A/IC} = r_1 \hat{\jmath}$, $\vec{\omega} = \omega \hat{k}$

$1.2 \, m/s \, \hat{\imath} = (\omega \hat{k}) \times (r_1 \hat{\jmath})$

$\hat{\imath}$) $1.2 \, m/s = -\omega r_1 \quad \therefore \quad \omega = \frac{-1.2 \, m/s}{0.15 \, m} = -8 \, \frac{rad}{sec}$ e.g. $\boxed{\vec{\omega} = -8 \, \frac{rad}{s} \, \hat{k}}$

b) To find velocity of upper rack, find velocity of pt B

IC at contact point: $\vec{V}_B = \vec{\omega} \times \vec{r}_{B/IC}$ w/ $\vec{r}_{B/IC} = (r_1 + r_2) \hat{\jmath}$

$= \left(-8 \, \frac{rad}{s} \hat{k}\right) \times (r_1 + r_2) \hat{\jmath}$

$= \left(8 \, \frac{rad}{s}\right)(0.15m + 0.1m) \hat{\imath}$

$\vec{V}_B = 2 \, m/s \, \hat{\imath}$

Point B is in contact with the upper rack, so $\boxed{\vec{V}_{upper\ rack} = 2 \, m/s \, \hat{\imath}}$

To find velocity of pt D, use IC at contact pt again

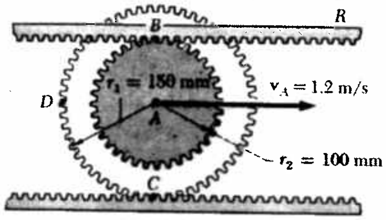
$\vec{V}_D = \vec{\omega} \times \vec{r}_{D/IC}$ w/ $\vec{r}_{D/IC} = (-r_1)\hat{\imath} + (r_1)\hat{\jmath}$

$= \left(-8 \, \frac{rad}{s} \hat{k}\right) \times (-r_1 \hat{\imath} + r_1 \hat{\jmath})$

$= \left(8 \, \frac{rad}{s}\right)(r_1)\hat{\jmath} + \left(-8 \, \frac{rad}{s}\right)(r_1)(-\hat{\imath})$

$\boxed{\vec{V}_D = 1.2 \, m/s \, \hat{\jmath} + 1.2 \, m/s \, \hat{\imath}}$

without using IC



SAMPLE PROBLEM 15.2

The double gear shown rolls on the stationary lower rack; the velocity of its center *A* is 1.2 m/s directed to the right. Determine (*a*) the angular velocity of the gear, (*b*) the velocities of the upper rack *R* and of point *D* of the gear.

a) no masses, no forces ⇒ try kinematics

The gear travels on the lower rack as it turns:

$$\text{distance traveled} = \underset{[\text{rad}]}{(\text{rotation angle})}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{2\pi r_1\,[\text{m}]}{1\text{ rev}}\right)$$

$$X_A = \frac{\cancel{2\pi} r_1 \theta}{\cancel{2\pi}} = r_1\theta$$

diff wrt time $\quad V_A = r_1\dot{\theta} = r_1\omega$

$$\omega = \frac{V_A}{r_1} = \frac{(1.2\,\text{m/s})}{(0.15\,\text{m})} = 8\,\frac{\text{rad}}{\text{sec}}$$

note that to move gear to right, $\vec{\omega}$ must be cw, so $\boxed{\vec{\omega} = -8\,\frac{\text{rad}}{\text{s}}\,\hat{k}}$

b) $\vec{V}_B = \vec{V}_A + \vec{\omega}\times\vec{r}_{B/A}$ w/ $\vec{r}_{B/A} = r_2\,\hat{j}$

$$= (1.2\,\text{m/s}\,\hat{i}) + (-8\,\tfrac{\text{rad}}{\text{s}}\,\hat{k})\times(r_2\,\hat{j})$$

$$= 1.2\,\text{m/s}\,\hat{i} + 0.8\,\text{m/s}\,\hat{i} = 2\,\text{m/s}\,\hat{i}$$

$$\boxed{\vec{V}_R = 2\,\text{m/s}\,\hat{i}}$$

$\vec{V}_D = \vec{V}_A + \vec{\omega}\times\vec{r}_{D/A}$ w/ $\vec{r}_{D/A} = (-r_1\,\hat{i})$

$$= (1.2\,\text{m/s}\,\hat{i}) + (-8\,\tfrac{\text{rad}}{\text{s}}\,\hat{k})\times(-r_1\,\hat{i})$$

$$= 1.2\,\text{m/s}\,\hat{i} + 1.2\,\text{m/s}\,\hat{j}$$

$$\boxed{\vec{V}_D = 1.2\hat{i} + 1.2\hat{j}\ \text{m/s}}$$