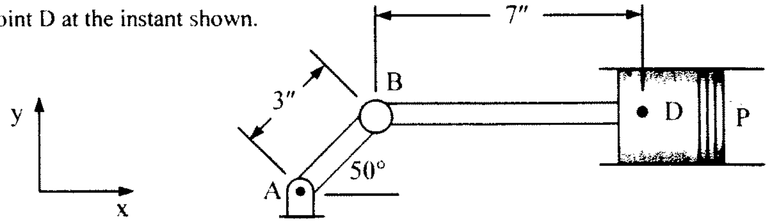
Example 4/12

Bar AB rotates with a constant angular velocity of 1600 rev/min counterclockwise. Determine the velocity of point D at the instant shown.



Note: No forces or masses given, it's probably kinematics

Strategy: get $\vec{v}_B$, then $\vec{v}_D$

Notice that rod AB is in fixed axis rotation, so

$$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/A} \qquad \text{b/c } A \text{ is the fixed axis so } \vec{V}_A = 0$$

$$= \left(1600 \tfrac{rev}{min}\,\hat{k}\right) \times \left(3in\left(\cos 50^\circ \hat{\imath} + \sin 50^\circ \hat{\jmath}\right)\right)$$

$$= \left(1600 \tfrac{rev}{min}\right)\left(\tfrac{2\pi rad}{1 rev}\right)\left(\tfrac{1 min}{60 sec}\right)\left(3in\right)\left(\cos 50^\circ \hat{\jmath} - \sin 50^\circ \hat{\imath}\right)$$

$$= \left(502.65\ ^{in}/_{sec}\right)\left(\cos 50^\circ \hat{\jmath} - \sin 50^\circ \hat{\imath}\right) = 323.1\,\hat{\jmath} - 385.1\,\hat{\imath}\ ^{in}/_{s}$$

Now we know $\vec{V}_B$, which is also on link BD

$$\vec{V}_D = \vec{V}_B + \vec{\omega}_{BD} \times \vec{r}_{D/B}$$

use $\vec{r}_{D/B} = 7in\,\hat{\imath}$ and $\vec{\omega}_{BD} = \omega_{BD}\,\hat{k}$

$$\vec{V}_D = \left(V_{BX}\hat{\imath} + V_{BY}\hat{\jmath}\right) + \left(\omega_{BD}\hat{k}\right) \times \left(7in\,\hat{\imath}\right)$$

$$V_D\,\hat{\imath} = V_{BX}\hat{\imath} + V_{BY}\hat{\jmath} + (7in)\,\omega_{BD}\,\hat{\jmath}$$

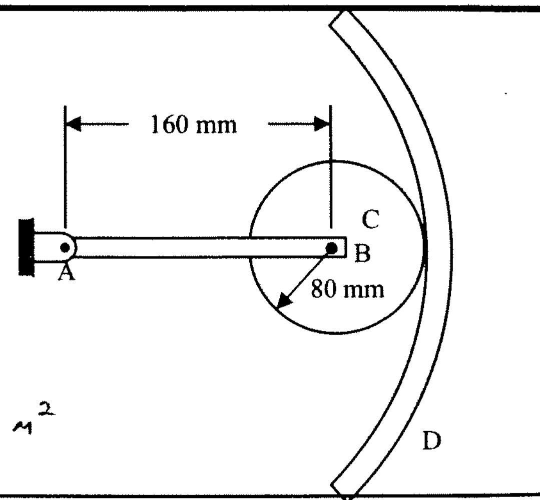
$\hat{\imath}$) $V_D = V_{BX}$ $\longrightarrow$ $\boxed{\vec{V}_D = -385.1\ ^{in}/_{s}\ \hat{\imath}}$

$\hat{\jmath}$) $0 = V_{BY} + (7in)\,\omega_{BD}$

$$\omega_{BD} = -46.16\ ^{rad}/_{sec}$$

Example 4/14

Gear C has a mass of 3.2 kg and a centroidal radius of gyration of 0.06 m. The uniform bar AB has a mass of 2.4 kg, and gear D is stationary (only part of gear D is shown). If the system is released from rest in the position shown, determine the equations necessary to solve for the angular velocities of gear C and bar AB after AB has rotated through 90°.



$$I_G\big|_{gear\,C} = (\text{mass})\left(\begin{matrix}\text{centroidal radius}\\ \text{of gyration}\end{matrix}\right)^2$$

$$= (3.2\text{kg})(0.06\text{m})^2$$

$$= 0.01152\ \text{kg m}^2$$

$$I_G\big|_{bar\,AB} = \left(\tfrac{1}{12}\right)(2.4\text{kg})(0.16\text{m})^2 = 0.00512\ \text{kg m}^2$$

Strategy: two positions ⇒ CoE $L = 160\text{mm}$, $r = 80\text{mm}$
kinematics

CoE ① = release, ② = vertical AB, sys = AB + disk C

$$E_{sys,1} = \underbrace{KE_{AB}}_{\to 0} + \underbrace{KE_C}_{\to 0} + m_{AB}\,g\underbrace{h}_{\to 0} + m_C\,g\underbrace{h}_{\to 0}$$

zero datum at horizontal

$$E_{sys,2} = KE_{AB} + KE_C + PE_{AB} + PE_C$$

$$KE_{AB} = \tfrac{1}{2} m_{AB} V_{GAB}^2 + \tfrac{1}{2} I_{GAB}\,\omega_{AB}^2$$

$$KE_C = \tfrac{1}{2} m_C V_{GC}^2 + \tfrac{1}{2} I_{GC}\,\omega_C^2$$

$$PE_{AB} = -m_{AB}\,g\,\tfrac{L}{2}$$

$$PE_C = -m_C\,g\,L$$

$W_{ext} = 0$ no slipping between gear C+D
no motion at pin A

$$E_{sys,2} - E_{sys,1} = W_{ext}$$

① $\tfrac{1}{2} m_{AB} V_{GAB}^2 + \tfrac{1}{2} m_C V_{GC}^2 + \tfrac{1}{2} I_{GAB}\,\omega_{AB}^2 + \tfrac{1}{2} I_{GC}\,\omega_C^2 - m_{AB}\,g\,\tfrac{L}{2} - m_C\,g\,L = 0$

Kinematics of AB – fixed axis rotation

② $V_{GAB} = \omega_{AB}\,\tfrac{L}{2}$

③ $V_{pt\,B} = \omega_{AB}\,L$

V_{GC} also

UNK
V_{GAB}
V_{GC}
ω_{AB}
ω_C

Kinematics C – IC is at contact point with D

④ $V_{GC} = \omega_C\,r$

solved!