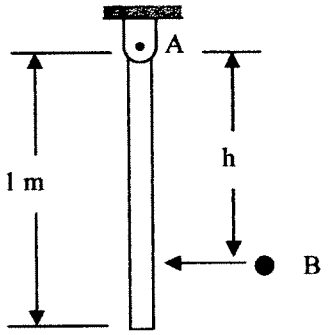
**Example 4/11**

A 50 gram ball traveling at 20 m/s strikes the homogeneous, long slender link with mass 1 kg, at a distance, h, below the pin joint A. The coefficient of restitution between the link and the ball is 0.5.

a) Find the distance h so that the pin reaction in the x-direction will be zero at the instant of impact.

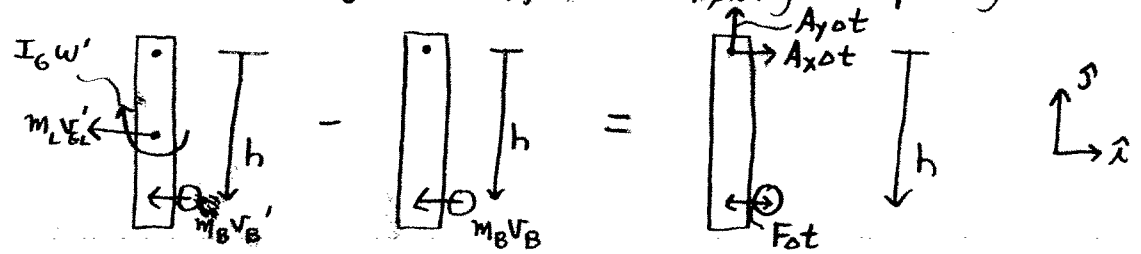
b) Plot the distance h as a function of the coefficient of restitution for $0<e<1$ and as a function of the initial velocity of B.

c) Plot the reaction force at point A in the horizontal direction as a function of h when e = 0.5.




This is very obviously an impact problem, so use $CLM_{F,I.}$ and $CAM_{F,I.}$

Draw a momentum diagram $\Pi_{sys,after} - \Pi_{sys,during} = Imp._{during}$




$CLM_{F,I.}$ (sys = rod + ball) $\hat{i}$

$$\Pi_{sys,after} - \Pi_{sys,before} = \Sigma Imp.\,\Delta t$$

$$\left[(-m_L v_{GL}') + (-m_B v_B')\right] - (-m_B v_B) = A_x \Delta t \qquad (1)$$

$CAM_{F,I.}$ (sys = rod + ball, pt A, $\circlearrowright$)

$$L_{sys,after} - L_{sys,before} = \Sigma M_{Imp}\,\Delta t$$

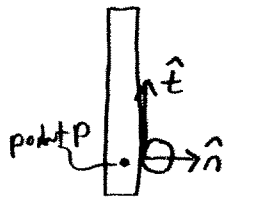
$$\left[I_G\omega' + m_L v_{GL}'\tfrac{L}{2} + m_B v_B' h\right] - \left[m_B v_B h\right] = 0 \qquad (2)$$

Kinematics to relate $\omega'$ & $v_{GL}'$

$$v_{GL}' = \omega'\left(\tfrac{L}{2}\right) \qquad (3)$$

CoR $\quad e(v_{PAn}^{\,0} - v_{PBn}) = v_{PBn}' - v_{PAn}'$ w/ A = rod, B = ball

note that




$$e(+v_B) = (-v_B') - (-v_{PA}') \qquad (4)$$

NOT $v_{GL}'$ !!!

use Kinematics $v_{PA}' = \omega' h$ (5)

Eqn ① $v_{GL}'$, $v_B'$ — Sunk, Seqns

② $\omega'$, $h$

③

④ $v_{PA}'$

⑤

Use MAPLE to solve using $A_{xat} = 0$

Conceptually, if $h \to 0$, $R_{xat}$ would act to the right

rod tries to ← and ↶,

$R_{xat}$ must resist both

if $h \to L$, $R_{xat}$ might act to the left

rod tries to ← and ↷

they may balance if $h > 4/3$

Extension - how high will the bar swing?

CoE ① ~~after impact~~, $E_{sys,1} = \frac{1}{2} m_L v_{GL}' + \frac{1}{2} I_G \omega'^2$

② at peak height, $E_{sys,2} = m_L g h_{swing}$

$W_{ext} = 0$ b/c $A_x$, $A_y$ act through zero distance