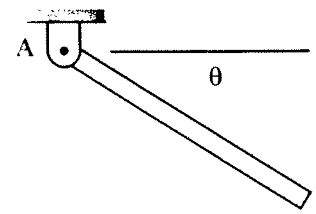
Example 4/7

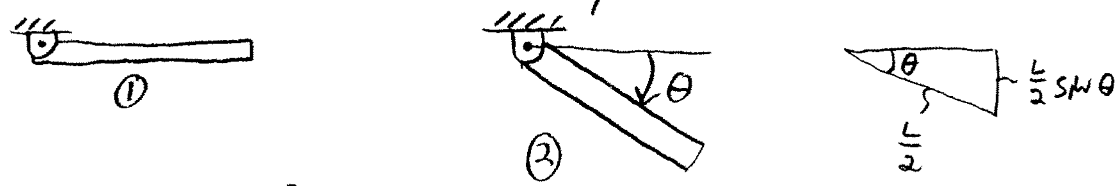
A bar of length L and mass m is released from rest at θ = 0.

a) Determine the equations necessary to find the reactios at point A as a function of θ.
b) Solve the equations you obtained in part a) for m = 2 kg and L = 0.5 m for θ = 0 to 180°



Approach: CLM_{rate}, CAM_{rate}, CoE

CoE from release at $\theta = 0°$ to arbitrary nonzero θ



$E_{sys,1} = \cancel{KE_1}^{\,0} + \cancel{PE_1}^{\,0}$ set height datum at pivot point
starts from rest

$E_{sys,2} = KE_2 + PE_2$

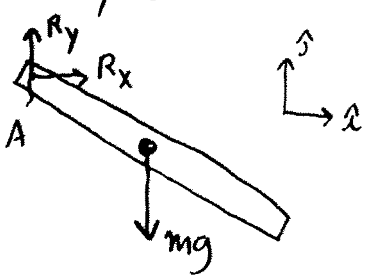
$= \left[\frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2\right] + \left[-mgh\right]$ $h = \left(\frac{L}{2}\right)\sin\theta$

$W_{ext} = 0$ since reaction forces move through zero distance and no applied Moments

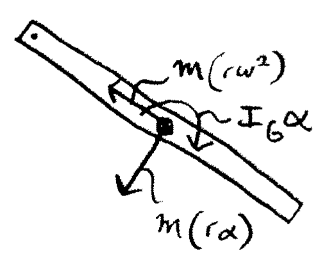
CoE $E_{sys,2} - E_{sys,1} = W_{ext}$

$$mg\frac{L}{2}\sin\theta = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 \qquad (1)$$

FBD at arbitrary θ



KD



No applied moments

If unsure which accels to include, put in both! Here I used

$a_t = r\alpha$ and $a_n = r\omega^2$

CLM rate (bar, $\hat{\imath}$) $\quad \frac{d}{dt}(\mathbb{P}_{sys,x}) = \sum F_x$

$$m\left[(-r\omega^2)\cos\theta - (r\alpha)\sin\theta\right] = R_x$$

$$m r\omega^2\cos\theta + m r\alpha\sin\theta = -R_x \qquad (2)$$

CLM rate (bar, $\hat{\jmath}$) $\quad \frac{d}{dt}(\mathbb{P}_{sys,y}) = \sum F_y$

$$m\left[(r\omega^2)\sin\theta - (r\alpha)\cos\theta\right] = R_y - mg \qquad (3)$$

CAM rate (bar, ↻, about COG)

$$\frac{d}{dt}(\vec{L}_{sys,G}) = \sum \vec{M}_G$$

only one term since we're taking moments about the center of mass

$$I_G\alpha(-\hat{k}) = (R_y)\left(\tfrac{L}{2}\cos\theta\right)\hat{k} - (R_x)\left(\tfrac{L}{2}\sin\theta\right)\hat{k} \qquad (4)$$

We have (1) v_G, ω

(2) R_x, α

(3) R_y

(4)

so we need another eqn, consider kinematics

if r is taken from the fixed axis pivot,

$$\underbrace{v_G = \omega\left(\tfrac{L}{2}\right)}_{\text{from } v = \omega r} \qquad (5)$$

END

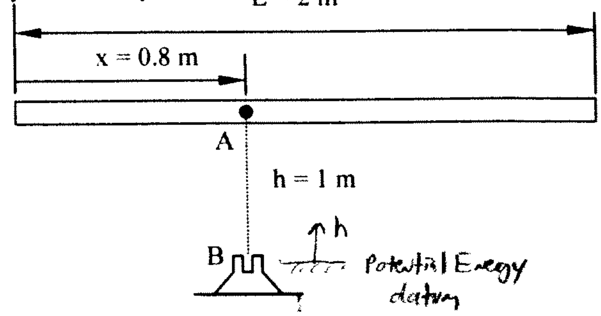
Example 4/9

The slender bar of mass m = 1.2 kg and length L = 2 m is released from rest in the horizontal position shown. If point A of the bar becomes attached to the pivot at B upon impact, determine immediately after the impact:

a) the angular velocity, ω, of the bar

b) the impulse exerted on the rod at A during the impact.

$I_G = \frac{1}{12} m L^2$ (Slender bar)



Approach: CoE from drop to just B4 impact with datum at catch point
$CAM_{F,T,}$ during impact

CoE $\quad E_{sys,1}\Big|_{drop} = mgh \qquad E_{sys,2}\Big|_{\substack{before\\hit}} = \frac{1}{2} m v_G^2 + \cancel{\frac{1}{2} I_G \omega^2}^{\,0}$

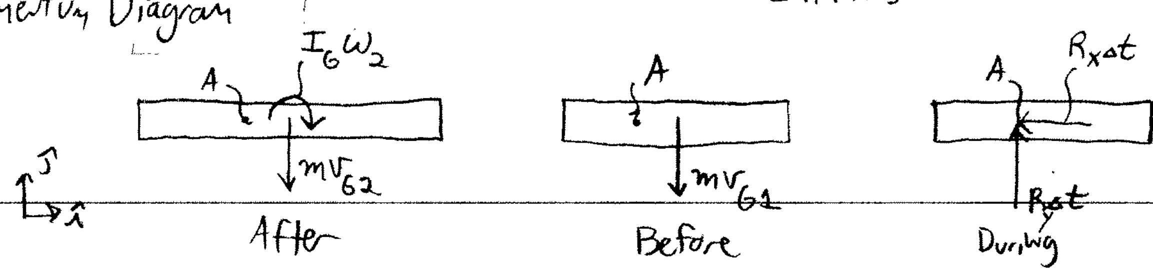
$W_{ext} = 0$ b/c no external forces during drop

$E_{sys,2} - E_{sys,1} = W_{ext}$

$\frac{1}{2} m v_G^2 = mgh \implies v_{G1} = \sqrt{2gh} \quad$ ①

$= 4.43\ ^m/_s$

Momentum Diagram



$CAM_{F,T,}$ (bar, ⨍, about pivot pt A)

$$\vec{L}_{\substack{sys,A\\after}} - \vec{L}_{\substack{sys,A\\before}} = \sum \vec{M}_{A\,\substack{\\during}} \Delta t$$

$$(I_G \omega_2) + (m v_{G2})(0.2m) - (m v_{G1})(0.2m) = 0 \qquad ②$$

Relate ω_2 to v_{G2} through Kinematics

$$v_{G2} = (\omega_2)(0.2m) \qquad ③$$

① + ③ into ② $\quad \left(\frac{1}{12}\right)(1.2\,Kg)(2m)^2 \omega_2 + (1.2Kg)(0.2m)\omega_2^{(0.2m)} - (1.2Kg)(4.427^{\,m/s})(0.2m) = 0$

$0.448\ \omega_2 = 1.0625 \quad \therefore \boxed{\omega_2 = 2.37\ ^{rad}/_s}$

To find impulse, use $CLM_{F.T.}$ (bar, $\hat{j}$)

$$P_{sys,2} - P_{sys,1} = \Sigma F_y \Delta t$$

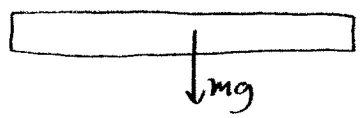
$$-m v_{G2} + m v_{G1} = R_y \Delta t$$

$$(-1.2\,kg)(3.37\,rad/s)(0.2\,m) + (1.2\,kg)(4.427\,m/s) = R_y \Delta t$$

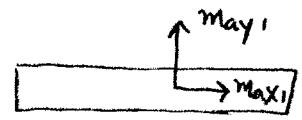
$$\therefore \boxed{R_y \Delta t = 4.744\ Ns \text{ upwards}}$$

Aside: what about CLM_{rate}? and CAM_{rate}? use to find α & Reactions after

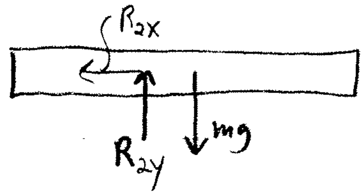
FBD_{before}



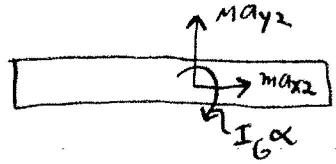
KD_{before}



FBD_{after}



KD_{after}



CLM_{rate} ($\hat{j}$, after) $\quad m a_{y2} = R_{2y} - mg \quad$ (10)

CLM_{rate} ($\hat{i}$, after) $\quad m a_{x2} = -R_{2x} \quad$ (11)

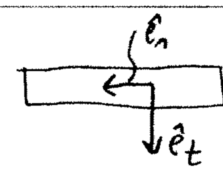
CLM_{rate} ($\hat{j}$, before) $\quad m a_{y1} = -mg \quad \therefore a_{y1} = -g$

CLM_{rate} ($\hat{i}$, before) $\quad m a_{x1} = 0 \quad \therefore a_{x1} = 0$

we already could have guessed this

if we find a_{y2}, then can get R_2

Recall $\hat{e}_t/\hat{e}_n$ kinematics $\quad \vec{a} = \underbrace{r\omega^2}_{-a_{x2}} \hat{e}_n + \underbrace{r\alpha}_{a_{y2}} \hat{e}_t$



from (11) $\quad R_{2x} = +mr\omega^2 = (1.2\,kg)(0.2\,m)(2.37\,rad/s)^2 = 1.348\ N \quad \leftarrow$ to the left

CAM_{rate} (COM, $\hat{k}$) $\quad I_G \alpha = R_{2y}(0.2\,m) \quad$ (12)

(10) using $a_{y2} = r\alpha$ gets $R_{2y} = mg + mr\alpha$ into (12)

$$\frac{I_G \alpha}{0.2\,m} = mg + mr\alpha$$

$$\alpha\left[\frac{I_G}{0.2\,m} - mr\right] = mg$$

$$\alpha = \frac{(1.2\,kg)(9.8)}{\left[\frac{0.4}{0.2} - (1.2)(.2)\right]} = 6.682\ rad/s^2$$

gets $R_{2y} = 13.36\,N$