Example 4/4

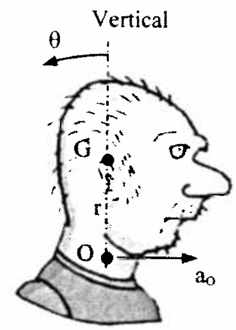
In an investigation of whiplash resulting from rear end collisions, rotation of the head is of primary interest. An impact test was performed and it was found that the angular acceleration of the head is

$$\alpha = 700\cos\theta + 70\sin\theta$$

Assuming the head is initially at rest,

a) Plot the angular velocity as a function of θ for $\theta = 0$ to $45°$

b) Determine the angular velocity of the head at $\theta = 45°$



Strategy: use kinematics

Solution:

$$\alpha = \frac{d\omega}{dt} = 700\cos\theta + 70\sin\theta$$

$$\downarrow$$

$$\omega\frac{d\omega}{d\theta} = 700\cos\theta + 70\sin\theta$$

$$\int_0^\omega \omega\, d\omega = \int_0^\theta (700\cos\theta + 70\sin\theta)\, d\theta$$

$$\tfrac{1}{2}\omega^2 = \left[700\sin\theta - 70\cos\theta\right]_0^\theta$$

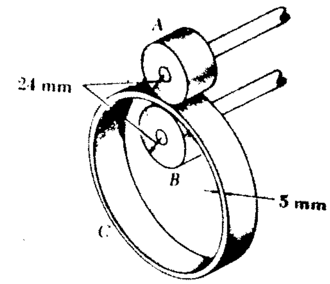
$$\tfrac{1}{2}\omega^2 = 700\sin\theta - 70\cos\theta - 70$$

↑ don't forget this!

$$\boxed{\omega = \sqrt{1400\sin\theta - 140\cos\theta - 140}}$$

15.20 Beer + Johnson

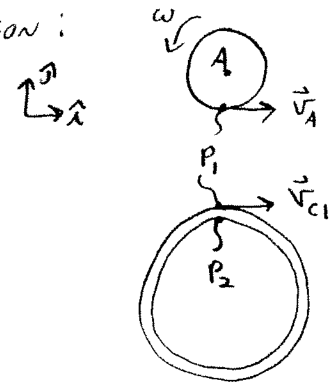
15.20 Ring C has an inside radius of 55 mm and an outside radius of 60 mm and is positioned between two wheels A and B, each of 24-mm outside radius. Knowing that wheel A rotates with a constant angular velocity of 300 rpm and that no slipping occurs, determine (*a*) the angular velocity of ring C and of wheel B, (*b*) the acceleration of the points of A and B which are in contact with C.



(Taken from Beer + Johnson)

Strategy : Kinematics

Solution :



$$\vec{v}_A = r\omega\,\hat{e}_t = r_A\omega_A\,\hat{\imath}$$

$$\vec{v}_{C_1} = \vec{v}_A = r_A\omega_A\,\hat{\imath}$$

also described by $r_{co}\,\omega_c\,\hat{\imath}$

$$\therefore\ r_{co}\,\omega_c = r_A\,\omega_A$$

$$\omega_c = \frac{r_A}{r_{co}}\,\omega_A = \frac{(24\,mm)}{(60\,mm)}\left(\frac{300\,rev}{min}\right)\left(\frac{1\,min}{60\,sec}\right)\left(\frac{2\pi\,rad}{1\,rev}\right)$$

$$\boxed{\omega_c = 12.57\ rad/sec \text{ or } 120\,rpm}$$

b) accel of P_1 on A $\quad \vec{a}_A = r\omega^2\hat{e}_n + r\cancel{\alpha}^{\,0}\hat{e}_t$

$$= r_A\omega_A^2\,\hat{\jmath} = (24\,mm)\left(\frac{300\,rev}{min}\right)^2\left(\frac{2\pi\,rad}{1\,rev}\right)^2\left(\frac{1\,min}{60\,sec}\right)^2\left(\frac{1\,m}{1000\,mm}\right) =$$

$$\boxed{\vec{a}_{AP_1} = 23.7\ m/s^2\ \hat{\jmath}}$$

accel of P_2 on B needs ω_B, similarly to above,

$$r_B\omega_B = r_{ci}\,\omega_c \quad \therefore\ \omega_B = 28.8\ rad/sec \text{ or } 275\,rpm$$

then $\quad \vec{a}_{BP_2} = r\omega^2\hat{e}_n + r\cancel{\alpha}^{\,0}\hat{e}_t$

$$= r_B\omega_B^2(-\hat{\jmath}) = \left(\frac{24\,mm}{1}\right)\left(\frac{1\,m}{1000\,mm}\right)\left(\frac{28.8\,rad}{sec}\right)^2(-\hat{\jmath})$$

$$\boxed{\vec{a}_{BP_2} = -19.9\ m/s^2\ \hat{\jmath}}$$