The resonant excitation of a wineglass using positive feedback with optical sensing

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We describe an apparatus that will sense the vibration of a wineglass through the jitter induced on a laser beam reflected from the glass wall. A positive feedback system provides a high level sound-wave-train that maintains the vibration of the glass, while a light-emitting diode lighting panel, also deriving its signal from the feedback system, allows the motion of the glass to be clearly observed in a user-controllable way. The positive feedback signal, along with observations from some additional experiments, can be used to highlight some of the nonlinear aspects of the resonance. Although the apparatus is primarily intended as a demonstration exhibit, we have found it useful also as a physics teaching aid.

I. INTRODUCTION AND OVERVIEW

Various types of apparatus to excite the resonance of a wineglass and ultimately to exceed its fracture limit have been developed over the past few years.[1–5] All of these have used the combination of a variable frequency signal generator and loudspeaker in some form to couple acoustic energy into the glass. The principal modification in our apparatus is that it gives the glass a fundamental role in its own excitation, by implementing it as part of a positive feedback loop. The use of feedback proves invaluable in the subsequent analysis of how the vibrating glass actually behaves as it tends toward its fracture threshold. The experimental study becomes especially intriguing when observations pertaining to the tapping of a glass are introduced, the result being a set of fascinating concepts geared toward modelling the associated physics of vibration, many of which have not been discussed previously in connection with this demonstration.

The key elements of the system we have built are shown in block diagram form in Fig. 1. A laser beam is reflected off the wall of a glass to sense its vibration. The jitter in the reflected light is converted to an electrical signal by directing the beam toward a position sensitive photodetector. This use of an optical lever to highlight the motion of the glass wall has been exploited previously as a viewing aid[6] with the use of a small mirror attached to the glass rim. Here, no such mirror was used, in order that the glass could vibrate freely. The reflected laser spot is thus rather diffuse, especially in the horizontal direction; however, this is not such a problem because the photodetector can be oriented so that it is sensitive to vertical motion of the reflected beam. The signal from the photodetector is fed through some processing electronics, including a narrow bandpass filter, to a power amplifier. The power amplifier feeds directly into a 300-W loudspeaker that in turn couples acoustic energy into the glass. A typical glass may have a fundamental resonance frequency of a few hundred Hz, giving the corresponding wavelength of acoustic waves from the loudspeaker to be of order 1 m. Thus, in order to keep the apparatus compact, we did not seek to set up an acoustical resonator to improve the coupling efficiency. The acoustic waves that excite the glass to vibrate are travelling waves; however, they have a fixed phase relationship with the glass wall, determined by the feedback electronics and the distance from the loudspeaker to the glass. Therefore, resonant buildup of the glass vibration is guaranteed, provided there is a sufficient sound pressure level (SPL) available. However, the high power levels quoted commonly for the amplifier and loudspeaker specification in this type of experiment should not be confused with the actual acoustic energy flux incident on the glass wall. To put things into perspective, we have found that thin-walled glasses shatter typically when SPL=125 dB at the glass, however, this amounts only to a fraction of 1 W of acoustic power over the glass wall.[7]

As well as provoking curiosity in the many interesting physical properties of a wineglass, most of which in some way stem from nonlinearities inherent in the system, it was equally important that the demonstration be of interest to a wide audience at an accessable level. To this end, our setup included three essential user-controllable features, depicted in Fig. 1 by the control input boxes A, B, and C. First, there is a gain control (box A) so that the motion of the glass could be sustained at a constant level, permitting study and comment during the demonstration. Second, the feedback was designed with an essentially narrow-band transfer function with a control included to enable tuning of the gain peak (box B). Finally, a novel lighting panel was developed in order that the motion of the glass could be clearly observed. This panel delivers flashes of illumination at the same frequency as the feedback signal, but with a periodic phase dither set by the user (box C). The complete system thus allows various vibrational modes of a glass to be selectively excited and viewed with only the simplest level of user intervention. A brief but comprehensive description of our apparatus will now be presented before we report on our observations and results.

II. THE POSITIVE FEEDBACK APPARATUS

In this section we will view our apparatus as a tool for the resonant excitation of a glass, presenting just enough detail to allow interested parties to reproduce our setup; if further details are required the principal author will be pleased to assist. The system is relatively inexpensive; indeed, all the components, excluding the power amplifier, can be obtained for significantly less than the cost of a signal generator with the necessary resolution to perform the same experiment.

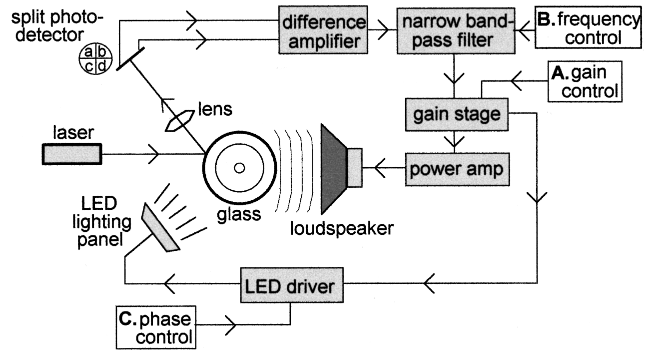

Fig. 1. Block diagram showing the experimental arrangement used to excite the resonance of the wineglass.

A. General design aspects

The entire setup was housed in a wooden box with acoustically absorbant leaded-foam lining to reduce the environmental noise pollution while the system is working. Some idea of the layout and scale of the system can be deduced from the photograph shown in Fig. 2 in which we have schematically overlayed some of the important features. The laser used is a key-ring laser diode pointer with $\lambda = 614$ nm providing around 5 mW. These devices are now available in many shops at small cost. Their compactness and relatively high power output make them the ideal choice for this application. The split photodetector is actually a quadrant photodiode with sectors that can be wired in adjacent pairs (ab and cd, or ac and bd in Fig. 1) to give the option of either vertical or horizontal sensitivity to motion of the reflected laser beam. We found that vertical sensing of the reflected beam offered the best results, partly because the curvature of the glass imposes horizontal divergence of the light, but also because of the insensitivity to mechanical sway of the laser and photodetector mounts that this orientation affords (both the laser and the photodetector are mounted on brackets that can slide up and down tall posts allowing a wide variety of glass shapes to be accommodated quickly and easily). To further improve the quality of the signal, a cylindrical lens was inserted to converge the horizontal band of reflected light into a very narrow strip at the photodetector. Glasses were retained in position by the use of two strong spring clips with rubber-padded jaws to press the base of the glass to the flat base of the box. If such a measure is not taken, the resonant excitation of the glass will be severely hampered due to losses encountered by mechanical energy propagating into the surroundings. A piece of leaded-foam with a cut-out is used to cover the front of the speaker and define an aperture smaller than the typical glass cross section. This better concentrates the sound on the nearest wall of the glass when using a comparatively large speaker.[1] The speaker was mounted such that most glasses could be placed with their rims level or just below the top of the aperture, with their wall being about 1 cm from the speaker diaphragm.

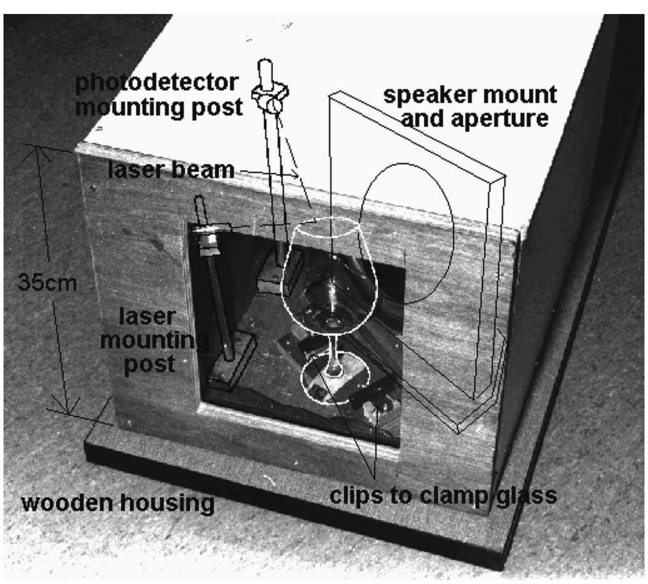

Fig. 2. Overlayed photograph giving some idea of the layout and scale of the apparatus.

B. Feedback electronics and vibrational mode selection

The feedback system electronics are detailed via the grouped stages in Fig. 3 which represent various key elements of the overall design, in a similar manner to the blocks in Fig. 1. First, the signals from adjacent halves of the photodetector are subtracted to give a position sensitive output. The switch S1 chooses between horizontal (h) and vertical (v) sensitivity to beam jitter. The balance indicator stage permits the user to coarsely align the laser beam reflected onto the photodetector and then to finely tune the difference signal electronically to within ± 20 mV of 0 V [when both red and green light-emitting diodes (LEDs) extinguish] using cross-wired 20-K variable load resistors.

The difference signal is then fed through a narrow band-pass filter with adjustable center frequency. This is a state-variable filter circuit with component values chosen using[8]

$$R_P = \frac{5\times 10^7}{f_P},\quad R_Q = \frac{10^5}{3.48Q + G - 1},\quad R_G = \frac{3.16\times 10^4 Q}{G}, \tag{1}$$

where f_P is the frequency of the gain peak, Q is the quality factor of the gain profile (f_P divided by the linewidth), and G is the gain at f_P. The values we have chosen enable the center frequency to be adjusted from around 100 Hz to several kHz by the use of R_P, which is calibrated beforehand so that different resonant modes of a glass can be more reliably selected. The linewidth of a high quality wineglass is less than 1 Hz for a resonance of several hundred Hz, and therefore the Q of this filter can be made reasonably high; we chose a value of around 10 here, which does not alter when f_P is varied as can be seen from Eq. (1). This stage is useful for selecting different modes of vibration of a given glass, while also suppressing the signals that arise as the result of low frequency structural vibrations associated with the apparatus housing, and laser, speaker, or photodetector mountings.

The next stage of the feedback is a preamplification circuit which contains the overall 10-K gain potentiometer. As we will see later, careful use of this control allows adjustment of the rate of change of glass vibration amplitude. The preamplifier drives a 300-W metal-oxide-semiconductor field-effect

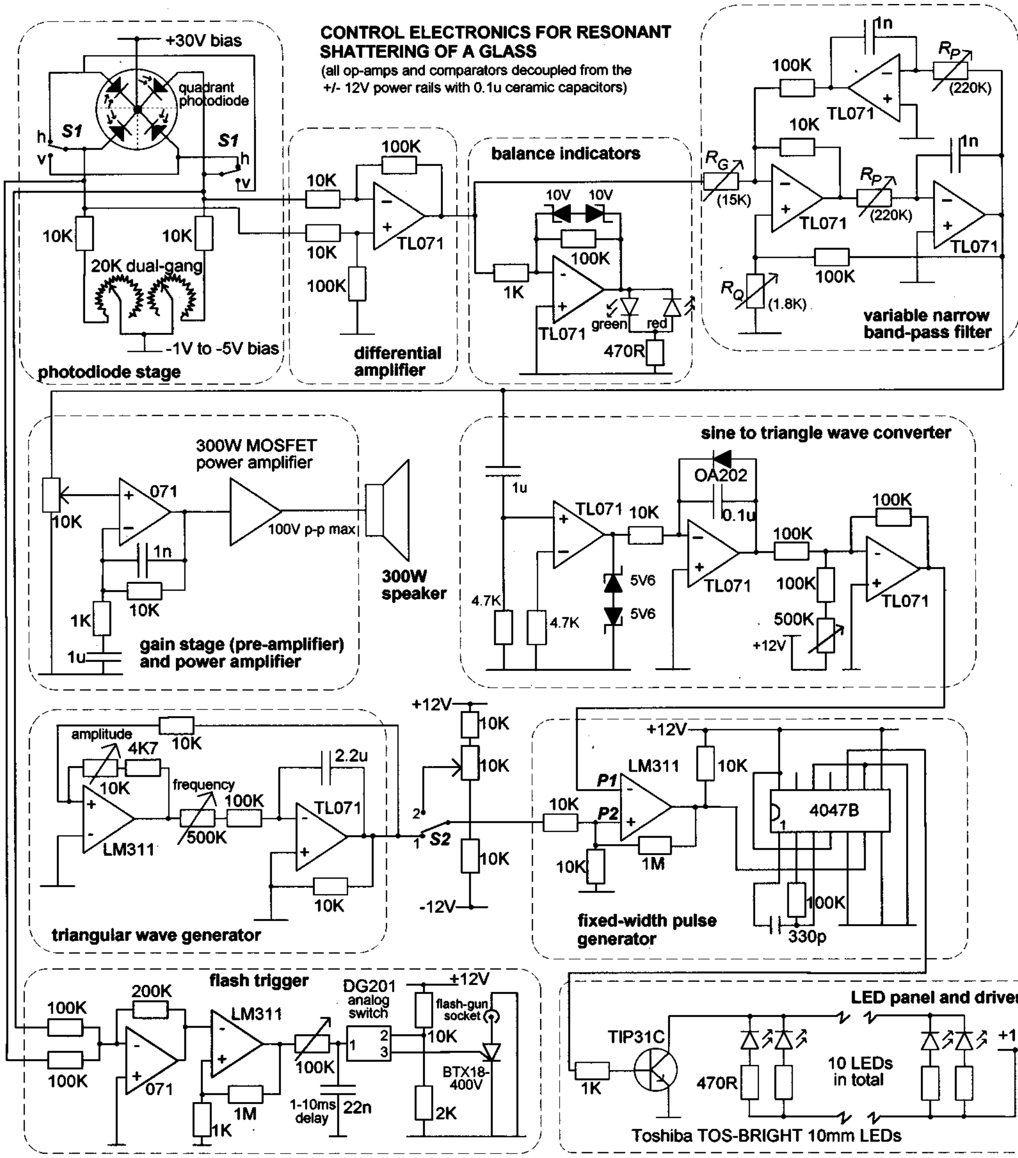

Fig. 3. Schematic diagram of the circuitry for the positive feedback system to excite and view the resonant vibration of a glass.

transistor (MOSFET) power amplifier that in turn drives the loudspeaker. The loudspeaker is an inexpensive car audio type, measuring 25 cm in diameter, and with 8-Ω impedance and 300-W peak power handling capacity.

C. Lighting panel

The amplitude of the feedback signal may vary considerably from one glass to another and also from one vibrational mode to another. Thus in order that the lighting panel work in all circumstances, the signal after the bandpass filter is converted into a fixed-amplitude triangular wave which is subsequently compared (at P1) with a reference signal (at P2) determined by the setting of S2. If S2 is in position 1, then this reference signal is a triangular wave of similar amplitude, but much lower frequency (adjustable from 0.5 to 10 Hz). In this case the output of the LM311 comparator stage is a square wave with varying duty cycle, the rising or falling edges of which provide the correct information to trigger the lighting panel with the appropriate phase dither to give the visual effect of low-frequency vibrational motion of the glass. If S2 is in position 2, then the reference signal is a static dc offset that can be adjusted using the 10-K variable resistor. This allows the relative trigger phase of the lighting pulse to be varied linearly giving the ability to change the apparent shape of the glass in ''real time.'' In either case, the triggering itself is achieved via the 4047B complementary metal-oxide semiconductor (CMOS) device which, in the implementation shown, generates pulses of fixed width (100 μs in this case) each time it detects a falling edge produced by the LM311 comparator. The pulse width is arranged so that the flash duration at the LED panel is long enough for the time-integrated apparent brightness to be reasonable, while also not being so long as to begin to blur the image of the vibrating glass. This balance is best achieved experimentally, and for glasses of considerably different resonant frequencies, it is better to adjust this parameter. The individual lighting elements selected here are 8-mm Toshiba TOS-BRIGHT LEDs which helped achieve a good compromise between pulse duty cycle and brightness.

D. Shatter capture photography

An additional feature that we implemented toward the end of the experiment allows the glass to be photographed at the point of fracture. This is achieved using a simple circuit containing an op-amp to sum the total photodetector current, a comparator, and an analog switch and thyristor. This arrangement senses the moment that the laser beam leaves the photodetector and triggers a standard photographic flash gun. A RC delay is included to allow some control over the instant of illumination. The 400-V thyristor is necessary here because many commercial flash units produce over 100 V at their camera interface sockets and cannot be triggered from low-voltage logic devices.

E. Analysis of operation

An important consideration with positive feedback systems is how to get them started from an initially zero input state. We can describe the evolution of the envelope of the vibration amplitude with time using

$$A(T)=A(0)+\frac{k-1}{\tau}\int_0^T A(t)dt, \tag{2}$$

where $A(t)$ is the envelope amplitude at time t, τ is the $1/e$ decay time or storage time of the glass, and k is the nominal open loop gain of the feedback system (including the coupling factor for acoustic pressure waves at the glass). Equation (2) is obtained by integrating the difference between the energy gained by the glass in time dt and that lost in dt due to the finite Q of the glass. Now if $A(t)$ is initially zero, the feedback process will not start. However, it was quickly found that a combination of seismic vibration and the acoustic spectrum at the loudspeaker associated with electronic noise in the circuitry produced an adequate input noise spectrum [in effect causing $A(0)\neq 0$] and initial excitation always took place automatically. Thereafter, adjustment of the bandpass filter frequency via R_P in Fig. 3 was all that was required to select various modes of resonance of the glass.

The complete positive feedback system (including the glass) has a response time which is modified by the loop gain k in Eq. (2) and is of order $\tau/(k-1)$. It is straightforward and rather instructive to write a small program to plot the response of the system based on Eq. (2), and doing this has proven useful for invoking better handling of the gain limiting control; this control is sensitive and too great an increase too quickly can cause the glass to excite to fracture threshold in a time much shorter than the natural storage time of the glass. In practice it needs to be adjusted very slowly past the critical point corresponding to $k=1$, so that the vibration can proceed slowly to the required value whereupon the $k=1$ operating state can be resumed. There were nonlinearities in the electronics and loudspeaker, particularly at high signal levels, and these produce acoustic harmonics which in principle could excite the glass also. However, we are fortunate here in that a wineglass, like a bell, has mode harmonics which are not integer multiples of the fundamental.[3,9]

III. EXPERIMENTAL INVESTIGATIONS AND RESULTS

The feedback apparatus allows considerably more control over the resonant excitation of a glass than does the traditional signal generator approach, allowing the study of various aspects of the vibration properties during the lead up to fracture; among these are the mode shape of the glass, the frequency and amplitude of vibration, and the coupling efficiency. In the next few sections we will report some interesting observations and measurements; however, first, let us briefly explain the reasons behind our choice of glassware.

A. Physical choice of the glass

By analogy with any resonant system, the vibrational amplitude for a particular mode will be amplified in proportion to the quality factor (Q) for that mode, and in general we should strive for glassware with as high a Q as possible. The Q of an average quality glass is usually determined by material and structural losses. However, measurements we have made with a sound pressure meter show that for thin-walled leaded glasses the mechanical loss becomes secondary to the acoustic radiation damping (the contribution to loss through viscous air damping is low). Radiation efficiency increases for higher vibration frequencies,[10] and therefore the Q should usually be expressed for a mode rather than the glass as a whole. It is instructive to note that, in contrast with the factors that define the Q of a glass, the fracture threshold depends strongly on microscopic defects in the glass which act as stress raisers for an applied force.[11] The emphasis on Q is merely to attain the required stress level in the glass by maximizing the proportion of k in Eq. (2) that comes from resonant coupling as opposed to electroacoustic gain.

When shopping around for glasses, the Q value can be judged in terms of the decay time of the glass vibrating in its fundamental tone ($1/e$ time for vibrational amplitude decay) via the simple relation

$$A(t)=A(0)e^{-\omega_r t/2Q}, \tag{3}$$

where $A(t)$ is the amplitude of vibration at time t, $\omega_r=2\pi f_r$ where f_r is the resonant frequency of the fundamental vibrational mode, and Q is related to the decay time τ by

$$Q=\pi f_r \tau. \tag{4}$$

However, when tapping glasses to estimate their quality, it is instructive to remember two points. First, the human hearing response is logarithmic and so the perceived sound will decay fairly linearly. Second, in view of Eq. (4), we should remember that a small glass with high f_r can ring for less time than a larger glass, but still have comparable or perhaps higher Q and thus generally afford the better choice. A good ear for music is certainly a useful attribute when selecting glasses, for then it is at least possible to attach a weighting to the frequency of the glass as well as the decay time.

One factor that strongly affects the Q of a glass is its shape. In our experience, medium-large wineglasses have exhibited the highest Q values (of order 3000–4000) and are easiest to excite to the fracture threshold. Large beakers are a good choice because of their large cross section with respect to the sound source, and their high Q, as was demonstrated by Walker[1] and also more recently exploited as a teaching aid/lecture demonstration.[12,4] However, they are expensive and comparable properties can be realized with large brandy glasses. Glassware that is too conical, like cocktail glasses, or too mechanically stiff, like low-circumference champagne flutes, do not offer high Q or good coupling characteristics for the incident sound waves and thus require much higher volume from the loudspeaker to cause significant excitation, let alone shattering.

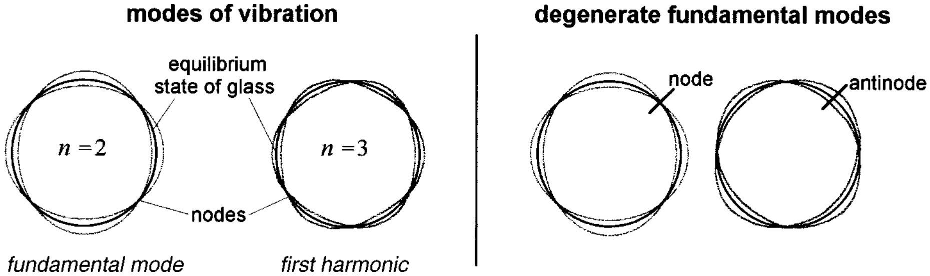

Fig. 4. The quadrupole and six-pole mode patterns that characterize the fundamental resonance of the glass and its first harmonic, and how the degenerate fundamental mode pair gives a static listener the impression of amplitude beats.

Most of the large wineglasses and brandy glasses that we used were made in the Czech Republic and obtained from department stores which import such glassware in bulk. Consequently, the glasses are relatively inexpensive at around $4 each. The brandy glasses are thin walled with a diameter at the rim measuring some 8 cm. They are 20 cm high and around half of the height is formed by the stem and the other half by the bulb. The wineglasses are about the same height, but have a rim diameter of 6 cm. Both types of glass had wall thickness around 1 mm, which varied only by a few percent over the top two-thirds of the bulb. The thickness below this point increases to around 2 mm near the stem. We have measured the resonance frequency for many of these glasses, and found the results all to be within a few Hz of 460 Hz for the brandy glasses and 630 Hz for the wineglasses. We also measured the Q of each glass by tapping their side and measuring the decay constant associated with the signal from a microphone placed near the rim and connected to a storage oscilloscope. Care has to be taken with this measurement because of a beat phenomenon associated with the vibrational mode of a glass, as described below. Partly for this reason, but also as a general cross check for the Q results, we used our apparatus to automatically excite the glass and measured the decay constant of the envelope of the decaying photodiode signal. (It might be thought that the Q could be estimated from the envelope of the increasing amplitude while the feedback was acting; however, as is evident from Eq. (2), the glass does not generally respond on the same time scale here, except for one value of loop gain given by $k=2$). The measured Q results ranged from 1800 to 2500 for the brandy glasses, and 3500 to 4200 for the wineglasses; however, for the reasons discussed above, the highest Q glasses are not necessarily those that were found to shatter the most readily.

B. Vibrational modes of the glass

If you take a random wineglass and tap its side near the rim, the chances are you will not hear a single tone decaying smoothly, but rather the sound will have a more complicated timbre due primarily to two processes. The first of these is excitation of higher order modes of resonance that generate tones of various frequency and decay times. The second effect is more subtle, observable as a relatively slow amplitude modulation of the tone from the glass, due to the excitation of pairs of degenerate modes where the modes in each pair differ slightly in frequency. This same beat effect is responsible for warble in church bells.[13–15] What we will refer to as the fundamental circumference mode shape and the next higher order mode are shown in Fig. 4, along with the degenerate mode pair for the fundamental mode. There is also the possibility of monopole and dipole circumference modes, corresponding physically to a breathing type stretching of the glass and a rigid-body swaying of the bulb; however, these are not predominantly observed in view of the extensional stiffness of the glass. We did observe swaying of glasses about their stems when struck, and this does create very low frequency dipole radiation, but this is usually in the subaudio range at around a few Hz or less.

Let us consider now what actually occurs when tapping the side of a glass. The wall near the point of impact undergoes a flexural displacement, and the motion during the subsequent return to equilibrium results in the propagation of traveling waves around the walls of the glass. The wave velocities corresponding to the various Fourier frequencies present in the initial impulse will differ because, by analogy with flexural waves in thin plates, the system is highly dispersive. Moreover, due to the cylindrical symmetry of the glass, each propagating wave traveling in a given direction round the glass will have a companion wave traveling in the opposite direction. The interference of the counter-traveling waves which have an integral number of wavelengths $n\lambda$ fitting in the circumference gives rise to the standing modes around the glass. The circumferential shape of these modes near the rim can be approximated by

$$A(\theta,t)=A_0[\sin(n\theta+v_F t)+\sin(n\theta-v_F t)], \tag{5}$$

where A_0 is the maximum amplitude of vibration and v_F is the velocity of the traveling flexural waves in glass. It should be noted that v_F will vary down the height of the glass bulb due to the wall thickness variation; however, as pointed out previously, the thickness is fairly constant over most of the upper section of the glass. Therefore the simplest way to deduce v_F, after Apfel,[16] is to think of the wall as a flat plate of thickness a, whereby the speed for flexural or bending waves of frequency ν_n is related to the speed for longitudinal sound waves v_L by

$$v_F=\left(\frac{\pi a v_L \nu_n}{\sqrt{3}}\right)^{1/2}. \tag{6}$$

The conditions $v_f=\nu_n\lambda$ and $2\pi r=n\lambda$ then yield

$$\nu_n=\frac{n^2 a v_L}{2\pi R^2\sqrt{12}}, \tag{7}$$

which shows that the frequencies of successive circumference modes do not ascend in integer multiples.[3,17] For example, we would expect the $n=3$ mode to have frequency 9/4 times that of the $n=2$ mode, and the $n=4$ mode to differ from the $n=3$ mode by 16/9. The accuracy of these ratios can be refined using the mode frequency results derived in the energy model analysis by French[17] which gives

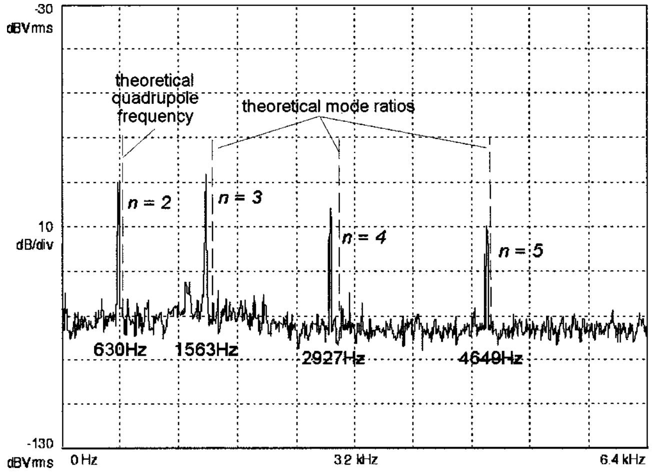

Fig. 5. Vibration spectrum for a wineglass with $a=1$ mm and $R=3$ cm showing the $n=2$, $n=3$, $n=4$, and $n=5$ modes of vibration. The $n=2$ frequency from French's energy analysis is plotted (650 Hz) along with the positions of the higher order modes referenced to the actual $n=2$ peak.

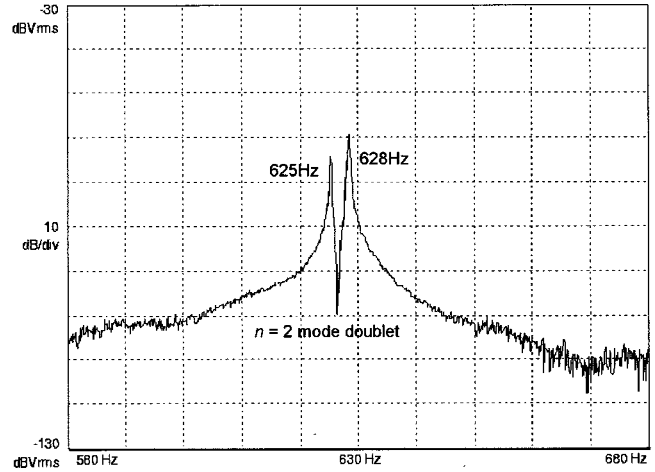

Fig. 6. Amplitude spectrum over a narrow frequency span measured using a microphone placed about 1 cm from the top of a medium-sized wineglass. The doublet structure of the degenerate quadrupole mode is clearly seen.

$$\nu_n = \frac{1}{12\pi}\left(\frac{3Y}{\rho}\right)^{1/2}\frac{a}{R^2}\left[\frac{(n^2-1)^2+(R/H)^4}{1+1/n^2}\right]^{1/2} \tag{8}$$

for the frequency of the nth circumference mode. In Eq. (8) Y and ρ are the Young's modulus and density of the glass ($\sim 6\times 10^{10}$ Pa and ~ 2.7 kg m^{-3}, respectively) and H is the height of the bulb section of the glass over which the thickness is fairly constant; the formula has been expressed for the lowest order vertical mode (no nodal circles between the bottom of the bulb and the rim). The mode structure for a real glass is shown in Fig. 5, which is based on the signal from a microphone placed near the rim of one of our wineglasses. The dotted lines superimpose the predicted position of the mode peaks, given the experimentally determined quadrupole frequency and then extrapolating to the $n>2$ modes using French's model with R/H approximated to unity. The agreement is quite impressive, and similar traces were recorded for several glasses. We did not measure any significant high order vertical vibrational modes for our glasses, although they almost certainly exist, particularly when the glasses are given a hard initial strike; by way of comparison, it is interesting to note how many high order modes can exist in a bell.[15,18–20]

The further properties of vibration become evident when examining one of the mode peaks over a much smaller frequency span. An example trace is shown in Fig. 6. Here, the $n=2$ quadrupole frequency is seen to be split, corresponding to the two degenerate quadrupole modes mentioned above. Both of these modes are equally valid solutions of the equations of motion of the vibration and could in principle be excited separately. However, in practice it was seen to be almost impossible to excite one mode alone, suggesting that they are not orthogonal but are coupled, perhaps as a result of nonlinearities in the equations of motion; evidence for nonlinear behavior will be presented later when we probe dependency between resonant frequency and vibration amplitude. The fact that the glass does not possess perfect cylindrical symmetry leads to slightly different responses in the glass for each mode, giving the two degenerate modes different resonant frequencies. There will be a periodic exchange of energy between the modes and the resultant $n=2$ tone produced by the glass will be significantly amplitude modulated at various points around the rim. The apparent audible beat frequency corresponding to Fig. 6 is 6 Hz, since there are two amplitude rises per $[1/(628-625)\mathrm{s}]$ period.

Another interesting consideration in view of Eq. (5) is the situation that occurs when the two counter-traveling waves have marginally different speeds. Then, the resulting mode pattern rotates, giving again the impression of amplitude modulation to a stationary listener. However, it is probably more likely that the nonuniformities around the glass wall will offer a preferential mode orientation where the vibration settles into a local minimum energy configuration, and so this additional effect is unlikely to be observed in practice. All of the above does suggest that, when identifying the best quality glasses, a useful secondary test to listen for over and above the basic decay time is the beat phenomenon, since the beat period will be greater for more uniformly blown glasses.

Although convincing visual evidence for the quadrupole mode pattern is obtained by viewing the vibration with the lighting panel in the feedback apparatus, we have also obtained acoustic information for the mode structure using two microphones placed at various points around the rim, as shown in Fig. 7. This experiment is an interesting additional study of the impulse response of a glass. As illustrated in the example traces of Fig. 7, the relative phase of the Fourier components between microphones 1 and 2 contains enough

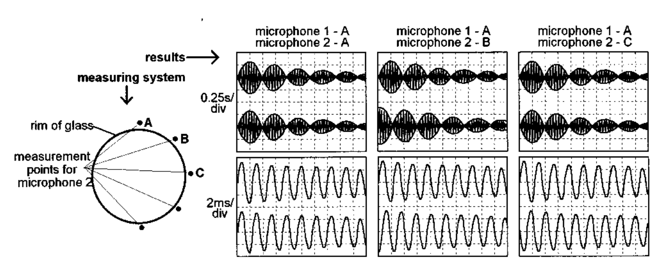

Fig. 7. Experimental arrangement for verifying the quadrupole vibrational mode shape and how it evolves after initial excitation of the glass. The traces show some example observations when the two microphones align with various antinodes of the $n=2$ degenerate modes (the traces have been cleaned of higher order mode distortions for clarity).

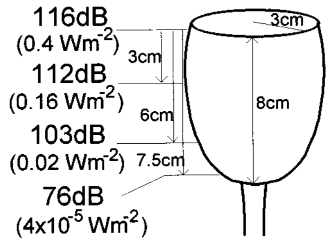

Fig. 8. The sound pressure levels and equivalent acoustic power fluxes measured immediately after striking the rim of a glass, exciting it with an initial amplitude of around 0.25 mm.

information to establish the mode shape, while the relative phase of the beat envelope allows orientation of the mode pattern to be established. However, one interesting feature that is not evident from this experiment, but which is revealed when illuminating a resonating glass with the lighting panel, is that the nodes are not truly stationary. As pointed out by Rossing,[21] the glass will move tangentially near the nodes as it resists extensional motion of its circumference, and this is clearly seen when adjusting the lighting panel to produce a fast (~10 Hz) apparent motion of the glass.

C. Coupling of energy to and from the glass

If a good quality glass is given a moderately hard tap it radiates sound from all parts of the bulb, but the highest SPL occurs near the rim. As an example, consider the measurements shown in Fig. 8 which relate to a wineglass struck halfway up the bulb with an initial amplitude at the rim of about 0.25 mm. We have measured the SPL and quoted the equivalent acoustic power at various points down the bulb of the glass. We may estimate the initial total power radiated to be around 10^{-3} W, where we have included a factor of order 2 because of the quadrupole nature of the vibration (not all parts of the glass radiate). Hence the energy lost per cycle is 10^{-3} W divided by the frequency of oscillation (around 630 Hz). The stored potential energy in the glass for the initial amplitude can be calculated using the energy analysis by French[17] to be around 10^{-3} J for the $n=2$ mode. Therefore, using the energy definition of the Q we can write

$$Q = 2\pi \frac{\text{energy stored per cycle, } E_s}{\text{energy lost per cycle, } E_l} \tag{9}$$

where $E_s \sim 10^{-3}$ J and $E_l \sim 1.6\times10^{-6}$ J for the initial cycle of vibration of the glass, which yields $Q \approx 3950$. This value for the Q is of the correct order but is too high; the actual Q value for the glass in Fig. 8 is closer to 3000. Even so, the calculation gives confidence that a significant, perhaps even the dominant, energy loss mechanism for the vibrating glass, is acoustic radiation damping. As a general guide, the glass can be made to vibrate at a given amplitude if the source acoustic wave pattern incident on it resembles the antenna pattern for radiation emission that the glass vibrating freely with the same initial amplitude would produce.[22] In practice, this is not achievable here if only because we are exciting only one side of the glass, whereas the glass radiates as a quadrupole for the $n=2$ mode. The SPL at the speaker when the glass has been resonantly excited to the same amplitude as in the impulse experiment above, was found to be around 120 dB (1 W m^{-2}). The time constant of the vibration amplitude growth during this excitation was also noted to be $\tau_g \sim 4$ s. The energy flux is incident over the entire speaker aperture which has area 2×10^{-3} m^2. The incident power is then 2×10^{-3} W. In comparison, the power radiated from the glass is of order 10^{-3} W. From this very rough estimate we deduce that there must be around twice the energy incident on the glass per cycle compared with the amount lost per cycle. The significance of the time constant noted above can now be examined. From Eq. (2) we can approximate τ_g to the value given by $\tau/(k-1)$ where k is the loop gain and τ is the amplitude decay constant for the glass; $\tau = 1.5$ s for the glass used here. We then have $k \sim 1.4$, which implies that the energy gained by the glass per cycle is around 1.4 times that lost per cycle. In view of the above, we deduce that around 1.4/2 (70%) of the energy from the speaker is absorbed by the glass. Given the approximations and uncertainties involved here, this number should not be taken too seriously, but it does illustrate that the energy coupling for the system when it is held exactly on resonance is much better than perhaps might be first thought. The fact that the energy absorbed by the glass per cycle is of the same order as the energy actually incident per cycle is an indication that the resonant action counterbalances the intrinsic poor coupling efficiency possessed by the system generally. However, a more interesting point to note is that the effective coupling efficiency is strongly dependent on the loop gain k, since we could have excited the same vibration amplitude above much more quickly by increasing the gain control. In practice, there is a limit set by the saturation threshold of the power amplifier and speaker.

Finally, we have observed that if the loop is broken and a signal generator output injected, then it takes much longer to excite the glass to its fracture threshold; such was our frequent experience when performing the demonstration manually. We could conclude that the basic experimental configuration should satisfy the antenna pattern matching criteria discussed above as far as possible, but then optimizing the coupling efficiency still further depends strongly on the success with which the resonance can be manually sustained. A principal reason for difficulty in tracking the resonance manually will be discussed next.

D. Shifting of the resonance

A glass that is forced into distortion will exhibit restoring forces that tend to change the glass shape back to its equilibrium state. For small deformations, this is essentially a linear process, the restoring forces being proportional to the deformation amplitude. However, for larger amplitudes of deformation, deviations from the linear model are observed, and in accordance with nonlinear oscillator theory, the free resonance frequency of the glass becomes a function of its amplitude. The nonlinearity stems not from the material stress-strain properties of glass, which can be regarded as Hookean to great precision,[23] but rather from the failure of approximations that are intrinsic to the simplified theories dealing with the small bending of an elastic plate or shell.[24–26] We have used a spectrum analyzer to observe the shifting in resonance frequency with increasing amplitude of vibration for several glasses. The plot in Fig. 9 is based on measurements of the feedback signal in our apparatus when exciting the $n=3$ mode of a chosen glass to four values of vibration amplitude. The measured amplitudes and frequencies are fit by the dashed curve in Fig. 9 while the damping of the oscillator manifests itself in the width of modified

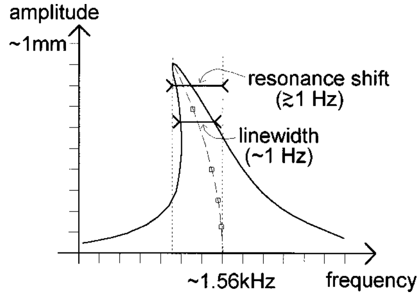

Fig. 9. An example curve for the forced vibration of a glass in the $n=3$ circumferential mode. The resonant frequency is found to decrease with increasing vibrational amplitude, suggesting nonlinearity of the softening type. The plotted points are based on measurements of the feedback signal, while the form of the response curve is qualitatively estimated based on the Q for the $n=3$ mode.

response curve, depicted by the solid line. The shifting downwards of resonance frequency with increasing amplitude is characteristic of a nonlinear softening of the vibrating glass. In other words, for higher amplitudes of deformation, the net restoring force toward equilibrium becomes less than that predicted by a perfectly linear theory. Such effects have been experimentally measured for forced flexural vibrations of a ring.[27]

By considering Fig. 9, it becomes apparent that if the glass resonance is to be tracked manually with a signal generator, then the correct procedure would be to slowly tune the frequency downwards through the resonance. If the frequency is tuned upwards, then the maximum resonant amplitude cannot be achieved in view of the unstable multivalued nature of the damped resonance.[28] The manual tracking is particularly necessary for high Q glassware. The linewidth of the response curve Δf is related to the Q for a particular mode via

$$\Delta f = \frac{f_r}{Q}, \tag{10}$$

where f_r is the resonant frequency of the mode. The Q of the $n=3$ mode (around 1500 here) is somewhat less than the Q for the $n=2$ mode, perhaps because the higher frequency modes are more efficient at radiating sound[10] and are truly more heavily damped, or may be due to mode coupling that instigates a gradual shift of energy into the quadrupole mode. Nevertheless, the shift in resonance frequency is still comparable with, or larger than, the linewidth of the resonance curve. For the fundamental $n=2$ mode, the shifts in resonance frequency as various glasses tended toward fracture ranged from 1 to 2 Hz. This has to be compared with typical linewidths of around only 0.25 Hz for this mode and therefore in the absence of feedback, the need for careful manual resonance tracking with high Q glassware is evident.

In passing, we should mention that the type of nonlinearity discussed above is not confined to relatively complex systems such as the vibrating glass. A much more familiar example is that of the simple pendulum. This also exhibits ''softening'' for large excursions from equilibrium, whereupon the approximate analysis, that applies only to the small angles of swing that define its ''simple'' region, would result in overestimation of the restoring force.

E. Resonant excitation and shattering of a glass

Finally we arrive at the inevitable climax of the vibrating glass demonstration; the shattering of the glass itself. This part of the experiment also offers much interesting physics to explore.

In order to comment on the actual fracture mechanisms themselves, we can refer to some historical studies on glass behavior[29,30] while making reference to the photographs in Fig. 10. It was interesting to note that the various glasses fractured at quite different vibrational amplitudes. Some could be made to resonate with a maximum peak to peak rim displacement of almost 1 cm while others would shatter when the rim deformed by not much more than 2 mm. The strength of an amorphous material such as glass is determined not by material properties but rather by the presence of structure defects. Such defects lower the stress threshold by amounts ranging over many orders of magnitude. Griffith[29] showed that the stress-raising ability of a microcrack depends on its characteristic length and radius of curvature of its tip. In particular, cracks with very small tip curvature can severely lower the strength of the material. An interesting example is that of glass A in Fig. 10 where we have deliberately introduced a scratch on the rim to try and influence its break point. However the crack has preferred to initiate at a defect somewhat displaced from the scatch, illustrating to what extent the natural microscopic defects can lower the fracture strength. Such is the wide variety of size and shape of defects that the vibrational amplitude at which a particular glass will fracture is impossible to predict; it certainly cannot be estimated from the Q. When a glass does shatter, the crack almost always begins at an antinode at the rim where the stress is greatest. The crack then propagates, using the elastic potential energy stored in the glass to rip apart the material bonds in its path. A recurring crack route seems to be a curved line beginning at the rim and running near the stem, perhaps as the crack follows a particular stress

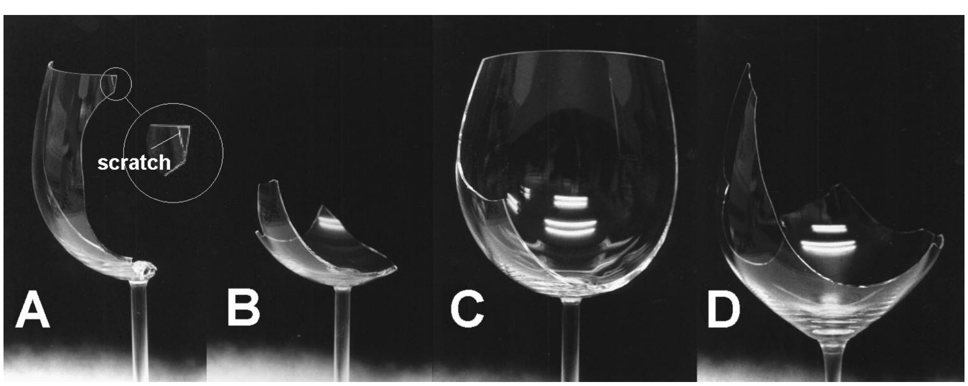

Fig. 10. Gallery of broken glasses photographed after being extracted from the apparatus. There is a frequent recurrence of a particular shape of curved crack propagation commencing at the rim.

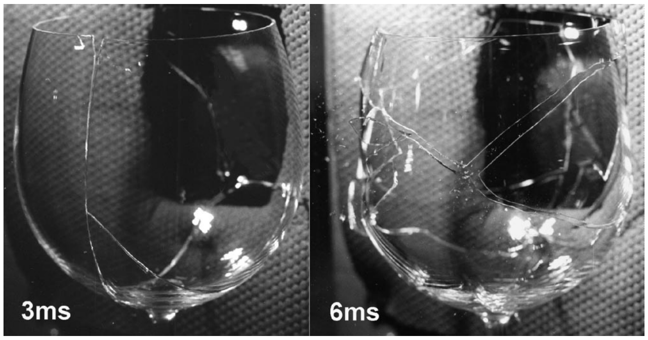

Fig. 11. Photographs capturing the shattering of two different glasses, where the delay between fracture and flash was set first at 3 ms and then 6 ms.

line associated with the mode shape. Some glasses show very clear branching of the running crack, for example, B, C and D in Fig. 10. It is thought that there is a maximum crack propagation speed, determined by the material properties of the glass, which is always less than the speed of sound in the material. If a crack approaches this terminal speed, the elastic energy is used to initiate branching rather than to increase the rate of progress of the original crack.[11]

The photographs in Fig. 11 were taken using the flash trigger circuit described in Sec. II. These show two brandy glasses each fractured after being excited in the $n=2$ mode, but showing different characteristics of the fracture. The position of the antinodes is defined by the speaker position, which can be seen in the background. It is clear that the first glass (captured 3 ms after the initial crack) has fractured at the rim near the foreground antinode, where the bending is locally maximal. The initial crack has thrown off a piece, before propagating at great speed round to the back of the glass. The pieces left after the crack has branched are quite large in this case, and this does not provide such a spectacular demise. We can refer again to French[17] to calculate the energy stored in this glass just at the point of fracture. Given that the rim vibrated with an amplitude of around 4 mm peak-to-peak just before fracture, we deduce the maximum stored potential energy to be 0.1 J. The energy which is not used to propagate the cracks is transferred primarily to kinetic energy in the ejected pieces. A more spectacular example is the 6-ms delay photograph in Fig. 11. Here, the glass has fragmented into many smaller pieces. The kinetic energy possessed by a piece can be estimated from its size and location in the photograph. There will also be some frictional loss, particularly when larger pieces slide against one another.

As a final observation, we have found that continuous excitation of the glass, even over a fairly short time, can cause a notable reduction in its strength. The experiment is fraught with chance, since we must be lucky to select a glass that will not shatter too readily. The chosen glass is excited to a fairly large vibrational amplitude, say 2 mm, and held at this level for a few seconds before being allowed to ring down freely. Next, the glass is excited at a much lower amplitude, around 0.5 or 1 mm and held at this level for over 1 min. When the glass is now excited slowly back to the original 2-mm displacement amplitude, there is a strong likelihood that it will shatter along the way. We have seen this effect when studying the motion of several glasses. The applied periodic stress may be opening up some of the surface defects over time, lowering the fracture threshold; however, the defects only propagate into running cracks when the available elastic energy is increased further.

IV. CONCLUSION

We have built a portable physics demonstration exhibit that automatically excites various resonant modes of a wide variety of glasses. The apparatus employs positive feedback, which not only affords considerable control over the excitation state of a glass but constitutes a tool with which to investigate the resonant properties of the glass. During the vibration, a high intensity LED lighting panel is driven indirectly by the feedback signal, providing convenient strobe effect illumination of the glass.

We have investigated various aspects of the physics; these have included the choice of a glass based on the mechanical Q value, the vibrational mode patterns supported, the beating between degenerate pairs of modes, the coupling efficiency of incident acoustic energy into the glass, the shifting in resonance frequency with changing amplitude of vibration, and the structural properties affecting the ultimate demise of the glass itself.

We have strived to explore as many avenues pertaining to the physics of the resonant system as possible, if only to stimulate further thought on this and similar processes which may appear to be deceptively straightforward on the surface. Indeed, although a principal objective of the apparatus was to stimulate curiosity in students who are learning physics, it is also notable, from discussion and reaction to the results, how practicing physicists can also be made to realize that there is more to this experiment than meets the eye.

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[5]C. Mazel, Classroom project description can be found at http://whiplash.stanford.edu/MIT/html/wineglass-resonance.html
[6]See the University of Melbourne demonstration apparatus web page at http:www.ph.unimelb.edu.au/lecdem/wd4.html
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