

Magnetic field of a dipole and the dipole–dipole interaction

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Abstract

With a data-acquisition system and sensors commercially available, it is easy to determine magnetic fields produced by permanent magnets and to study the dipole–dipole interaction for different separations and angular positions of the magnets. For sufficiently large distances, the results confirm the $1/R^3$ law for the magnetic field and the $1/R^4$ law for the interaction force between two dipoles, as well as their angular dependences.

1. Introduction

The magnetic field of a dipole and the dipole–dipole interaction are considered in many textbooks (e.g., Cheng (1992), Chabay and Sherwood (1995), Griffiths (1999)). The magnetic field produced by a magnetic dipole at a distance R much larger than the size of the dipole (the so-called dipole approximation) is given by

$$\mathbf{B} = (\mathbf{a}_r 2 \cos \theta + \mathbf{a}_\theta \sin \theta) \mu_0 m / 4\pi R^3, \quad (1)$$

where μ_0 is the magnetic constant, m is the magnetic moment of the dipole, θ is the angle between the dipole axis and the direction to the point where the field is measured, and $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are the unit coordinate vectors (radial and tangential). The magnetic field of the dipole is inversely proportional to R^3 and generally contains both radial and tangential components.

The force between two dipoles having the same magnetic moments m and their axes aligned equals

$$\mathbf{F} = \pm \mathbf{a}_r 3\mu_0 m^2 / 2\pi R^4. \quad (2)$$

The interaction force is thus inversely proportional to R^4 . On the other hand, when the distance between the poles of two interacting bar magnets is much smaller than their length, the influence of the far poles may be neglected. In this case, the interaction force is inversely proportional to the distance squared.

Many papers have been devoted to the magnetic field of a dipole and the dipole–dipole interaction. Lufburrow (1963) measured the interaction force between poles of two bar

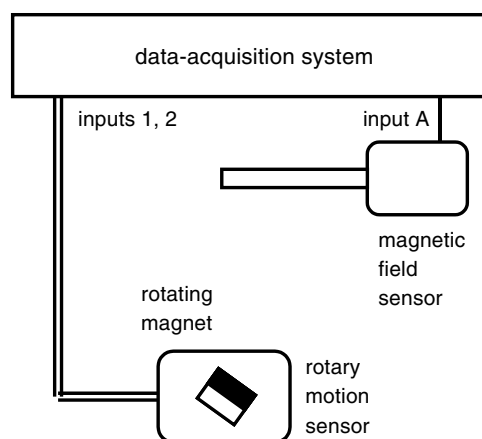


Figure 1. Diagram of the setup for measuring the magnetic field of a permanent magnet.

magnets with a balance. Romer (1973) presented a modified version of this apparatus. Defrancesco and Zanetti (1983) determined the magnetic repulsion between two magnets. Levin (1984) measured the magnetic moment of a compass needle. For this purpose, one has to determine the period of free oscillations of the needle in a known magnetic field and its moment of inertia. Shelton and Kettner (1988) measured the interaction force and potential energy for a pair of magnetic pucks. Bisquert *et al* (1990) analysed small oscillations of a bar magnet along the axis of a circular current-carrying coil. The period of the oscillations depends, in particular, on the dipole moment of the magnet. Lukefahr (1992) studied the dipole–dipole interaction with an air track. Mc Tavish (2000, 2001) calculated the field pattern due to a magnetic dipole.

Recently, Castañer *et al* (2006) reported on measurements of the magnetic dipole interaction using spring dynamometers. With this very simple technique and a magnetic field meter, the authors measured the attractive and repulsive forces between two similar permanent magnets, the magnetic field of the magnet at various distances, and the torque acting on the magnet in a homogeneous magnetic field. From the data, the magnetic moment and the residual induction of the magnets were evaluated.

The aim of this paper is to show how to measure, with a data-acquisition system and commercially available sensors, the magnetic field of a magnetic dipole and the dipole–dipole interaction, including their angular dependences. We employ the *ScienceWorkshop* data-acquisition system with *DataStudio* software from PASCO Scientific. Three PASCO sensors are used, namely the *magnetic field sensor* (CI-6520A), *rotary motion sensor* (CI-6538) and *force sensor* (CI-6537). For laboratory experiments and lecture demonstrations employing this data-acquisition system, see the recent manual by Kraftmakher (2007).

2. Magnetic field of a magnetic dipole

A strong ceramic permanent magnet ($12 \times 12 \times 12 \text{ mm}^3$ in size) is used in the measurements. Such magnets provide a significant dipole moment. Due to their small dimensions, the dipole approximation remains valid at relatively short distances. The magnet is attached to the shaft of the *rotary motion sensor* (figure 1). Its magnetic field is measured by the *magnetic field sensor*. For various distances, R , from the magnet, both components of the field (radial and tangential) are measured in turn.

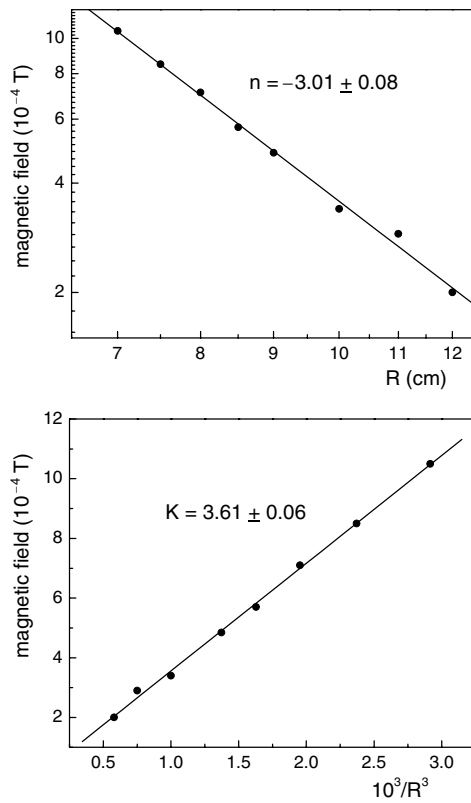


Figure 2. Log–log plot of the axial magnetic field versus R and plot of the magnetic field versus $1/R^3$.

To exclude possible errors caused by other magnetic fields and by the drift of the sensor, the data are recorded during one full rotation of the magnet. The magnetic field B is taken as half of the difference between the maximum and minimum values recorded. When presenting the data as $\log B$ versus $\log R$, the slope of the graph is close to the theoretical value $n = -3$ (figure 2). The data are in good agreement with equation (1): (i) the radial field is twice the tangential field (figure 3), and (ii) both fields are inversely proportional to R^3 . According to equation (1), the slope of the graph of B versus $1/R^3$ should equal $K = \mu_0 m / 2\pi$. From the slope obtained, the dipole moment of our magnet is 1.80 A m^2 . For a uniformly magnetized magnet, the residual induction B is related to the dipole moment and the volume V of the magnet: $B = \mu_0 m / V$. In our case, $B = 1.3 \text{ T}$.

3. Dipole–dipole interaction

Two similar ceramic permanent magnets ($12 \times 12 \times 12 \text{ mm}^3$ in size) are used in measurements of the interaction force between them. One of the magnets is fixed to the *force sensor*, and the second is attached to the shaft of the *rotary motion sensor* and positioned at a distance, R , just below the fixed magnet (figure 4). The *force sensor* measures the vertical component of the force acting on the first magnet. This means that the data obtained relate to the axial component of the interaction force. To exclude possible errors caused by other magnetic

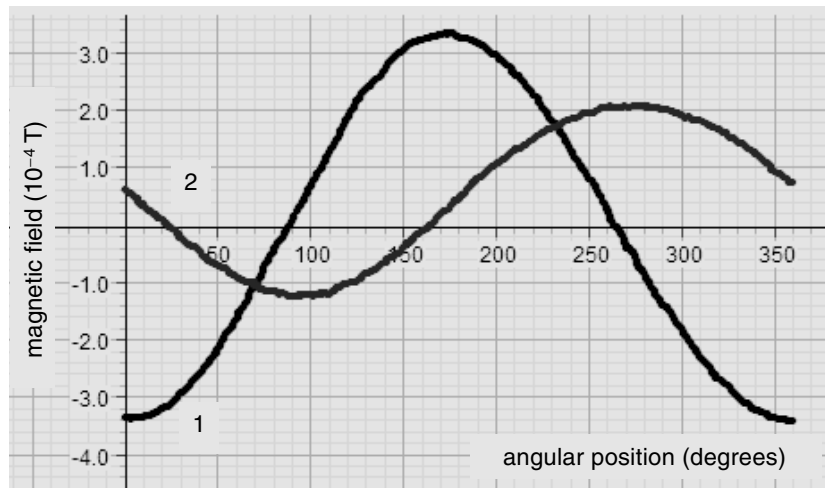


Figure 3. Angular dependence of the (1) axial and (2) tangential components of the magnetic field, for $R = 10$ cm.

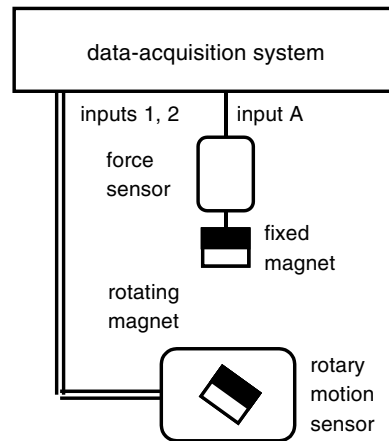


Figure 4. Diagram of the setup for measuring the dipole–dipole interaction between two magnets.

fields and by the drift of the sensor, the data are recorded during one full rotation of the magnet. The interaction force F is taken as half of the difference between the maximum and minimum values recorded, that is, as a mean value of the attractive and repulsive forces.

When presenting the data as $\log F$ versus $\log R$, the slope of the graph is close to the theoretical value $n = -4$ (figure 5). According to equation (2), the slope of the graph of F versus $1/R^4$ should equal $K = 3\mu_0 m^2/2\pi$. From the slope obtained, the dipole moment of each magnet equals 1.69 A m^2 , in reasonable agreement with that from the measurements of the magnetic field. With this value of the dipole moment, $B = 1.2 \text{ T}$.

Data obtained during rotation of the second magnet can be presented in polar coordinates (figure 6). For this purpose, it is necessary to exclude an offset of the data and replace the force by its absolute value. The polar diagram is close to what is expected.

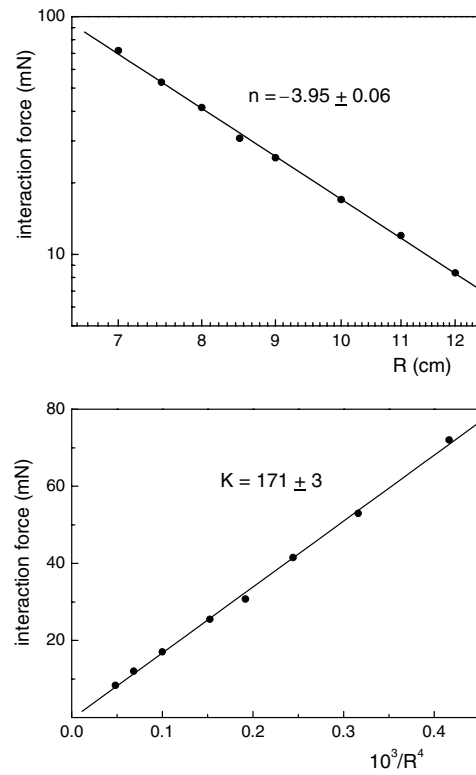


Figure 5. Log–log plot of the interaction force versus R and plot of the force versus $1/R^4$.

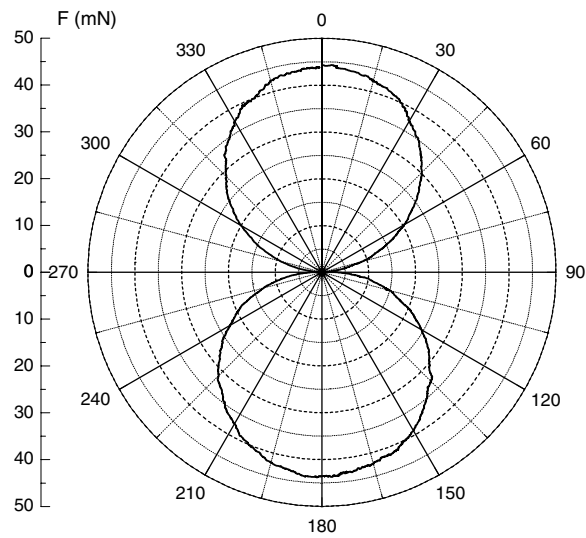


Figure 6. Polar diagram of the interaction force, for $R = 8$ cm.

With the sensors used, it is easy to perform measurements for other arrangements of the two magnets, as was proposed by Castañer *et al* (2006). Moreover, the angular dependences of the interaction forces are measurable.

4. Conclusion

The dipole–dipole interaction rapidly decreases with increasing distance between the dipoles. However, measurements of this interaction may become a routine task with a data-acquisition system and sensors now commonly available. An advantage of this approach is that the magnetic fields and interaction forces, including their angular dependences, can be measured in a short time.

References

- Bisquert J, Hurtado E, Mafé S and Pina J 1990 Oscillations of a dipole in a magnetic field: an experiment *Am. J. Phys.* **58** 838–43
- Castañer R, Medina J M and Cuesta-Bolao M J 2006 The magnetic dipole interaction as measured by spring dynamometers *Am. J. Phys.* **74** 510–3
- Chabay R and Sherwood B 1995 *Electric and Magnetic Interactions* (New York: Wiley)
- Cheng D K 1992 *Field and Wave Electromagnetics* 2nd edn (Reading, MA: Addison-Wesley)
- Defrancesco S and Zanetti V 1983 Experiments on magnetic repulsion *Am. J. Phys.* **51** 1023–5
- Griffiths D J 1999 *Introduction to Electrodynamics* 3rd edn (Upper Saddle River, NJ: Prentice-Hall)
- Kraftmakher Y 2007 *Experiments and Demonstrations in Physics. Bar-Ilan Physics Laboratory* (Singapore: World Scientific)
- Levin E 1984 Magnetic dipole moment measurement *Am. J. Phys.* **52** 248–50
- Lufburrow R A 1963 Inverse-square law experiment *Am. J. Phys.* **31** 60–2
- Lukefahr H G 1992 Magnetic dipole interactions on an air track *Am. J. Phys.* **60** 1134–6
- Mc Tavish J P 2000 Field pattern of a magnetic dipole *Am. J. Phys.* **68** 577–8
- Mc Tavish J P 2001 Field pattern of a magnetic dipole *Am. J. Phys.* **69** 1112 (erratum)
- Romer A 1973 Magnetic repulsion: an introductory experiment *Am. J. Phys.* **41** 1332–6
- Shelton D P and Kettner M E 1988 Potential energy of interaction for two magnetic pucks *Am. J. Phys.* **56** 51–2