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# Understanding the magnetic susceptibility measurements by using an analytical scale

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#### Abstract

A description of the measurement procedure, related theory and experimental data analysis of the magnetic susceptibility of materials is given. A short review of previous papers in the line of this subject is presented. This work covers the whole experimental process, in detail, and presents a pragmatic approach for pedagogical sake.

### 1. Introduction

The physical properties of matter are quantified by a set of numbers, the characteristic parameters of each substance. In the case of magnetic interactions, the materials have one of the three properties: diamagnetisms, paramagnetism or ferromagnetism. The specific presence in the matter of one of these characteristics depends on the number of paired electronic spins in the sample. In particular, the diamagnetism is originated by the magnetic orbital moment induced by the application of an external magnetic field with intensity  $\vec{H}$ . The paramagnetism and ferromagnetism have their origins in the averaged alignment of the spin magnetic moment in the direction of  $\vec{H}$  [1–4]. The quantification of the response of the matter in terms of these three properties is named the magnetic susceptibility ( $\chi$ ), which is a dimensionless constant. Nevertheless, the experimental statement of the magnetic nature of a sample is not an easy task. In general, the measurement of this quantity implies the determination of the components of a second rank tensor [2, 3].

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**Table 1.** Factor conversions in different expressions of the magnetic susceptibility from cgs to the SI unit system,  $\rho$  is the density of the substance in kg m<sup>-3</sup> and  $W_a$  is the molar mass in kg mol<sup>-1</sup>.

Susceptibility name	Symbol	Equation	SI Units	cgs/SI
Bulk	$\chi$ or $\chi_{\nu}$	M/H	Dimensionless	$1/4\pi$
Mass	χρ	χν/ρ	$m^3 kg^{-1}$	$10^{3}/4\pi$
Molar	Ҳм	$\chi_{v}W_{a}/\rho$	$M^3 mol^{-1}$	$10^{6}/4\pi$

From the thermodynamics point of view,  $\chi$  is the ratio of the magnetization  $\vec{M}$  of a sample under the influence of an external magnetic field  $\vec{H}$ , where the field intensity tends to zero [5–7], that is

$$\chi = \lim_{|\Delta \overrightarrow{H}| \to 0} \frac{|\Delta \overrightarrow{M}|}{|\Delta \overrightarrow{H}|}.$$
(1)

If we deal with linear and isotropic materials, a magnetization proportional to the external magnetic field appears, and then, the second rank tensor  $\chi$  is reduced to

 $M = \chi H. \tag{2}$ 

This expression is usually employed in the assessment of the magnetic susceptibility. It must be pointed out that the magnitude of  $\chi$  depends on the system of units used. For instance, it is possible to find  $\chi$  in chemical handbooks, with cgs units. Table 1 shows the factor conversion to SI for volumetric, mass and molar  $\chi$ .

On the other hand, electromagnetism and thermodynamics texts, for basic and advanced students, deal with the magnetic properties of matter. Nevertheless, there is a lack of information about experimental developments intended to perform measurements of the magnetic properties, especially  $\chi$ , although in a few specialized books some techniques to quantify the magnetic susceptibility can be found [8]. Also, a couple of methods using a weighing scale were reported in 1993 by Davis [9].

Because analytical scales are easily found in teaching and research laboratories, their use is one of the most versatile and economic procedures to measure magnetic susceptibility. They can be used with solids or liquids without any problem. Nevertheless, this is not a measurement modality which is widely available in an undergraduate science laboratory. The description of this technique is usually presented in research level papers or texts [10–12] that are not pedagogical in order to be used as a laboratory practice.

This paper presents the necessary stages to successfully undertake the experimental setup of this technique to measure  $\chi$  in solids and liquids. Special emphasis is given to the determination of the size and geometry of the samples.

# 2. Theory

Any substance is magnetized when exposed to a magnetic field. If the magnetization vector points in the same direction of the external field, the material is called paramagnetic or ferromagnetic and diamagnetic if the direction is opposite. In any case, the magnetized sample becomes a new permanent or induced magnet and then it exerts a force of attraction or repulsion on the external source of the magnetic field. This magnetic force was, in this study, detected with an analytical scale, as shown in figure 1. The magnetic force close to a magnetic dipole  $\vec{m}$ , in the presence of an external non-uniform magnetic field is [8]

$$\vec{F} = (\vec{m} \cdot \vec{\nabla})\vec{B}.\tag{3}$$



balance plate

Figure 1. Schematic diagram of the experimental setup, showing the main components.

On the other hand, the magnetic induction  $\vec{B}$ , in a point *P* indicated by the position vector  $\vec{r}$ , due to a dipole with the magnetic moment  $\vec{m}$ , is given by the expression

$$\vec{B} = \frac{\mu_0}{4\pi r^3} [3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m}],$$
(4)

where the constant  $\mu_0/4\pi = 10^{-7}$  N A<sup>-2</sup> is the magnetic permeability and  $\hat{r}$  is an unitary vector along the direction of  $\vec{r}$ , which goes from the dipole position to *P*. If  $\vec{m}$  and  $\vec{r}$  are along the *z*-direction, then  $\vec{m} \cdot \vec{r} = m_z$  and we just deal with the component of  $\vec{B}$  along *z*, that is, equation (4) reduces to

$$B_Z = \frac{\mu_0 m_z}{2\pi z^3}.$$
(5)

Equation (3) is valid where a magnetic moment is induced in the matter and moreover when the magnetic induction has the form of equation (4) or (5). These arguments are essential to obtain the model for the magnetic susceptibility of matter in the presence of a dipolar magnetic field.

#### 2.1. Model I: solution to a semi-infinite medium

In order to model the interaction force between the sample and the magnet, we assume that the sample is an infinite plane in which, in this case we deduce the force from equation (3). The typical image method proposed by Davis [9], is used, considering the system formed with the original dipole surrounded by air in front of an infinite and magnetizable plane (the sample). Then, an image dipole of the magnetic moment  $\vec{m}_i$  is introduced to replace the sample simplifying the calculation. If we assume that the plane z = 0 matches the surface of the sample,  $\vec{m}$  is the magnetic moment of the dipole which magnetizes the sample which is placed at a distance d under the plane along the z-axis, then, both the real and image dipoles are separated by a distance 2d. So, the z-component in equation (3) leads to

$$F_z = m_i \left[ \frac{\partial}{\partial z} \left( \frac{\mu_0 m}{2\pi z^3} \right) \right]. \tag{6}$$

This equation, when evaluated at z = 2d, becomes

$$F_z = 3m_i \frac{\mu_0 m}{32\pi d^4}.$$
 (7)

In order to find the relationship between m and  $m_i$  (the z-components of  $\vec{m}$  and  $\vec{m}_i$ , respectively) we apply boundary conditions on the surface of the sample. Analysing a small volume enclosing the boundary between the two media, the continuity conditions can be written as (provided that the current density j = 0)

$$(\vec{B}_1 - \vec{B}_2) \cdot \hat{n} = 0, \tag{8}$$

$$(\vec{H}_2 - \vec{H}_1) \cdot \hat{t} = 0, \tag{9}$$

where  $\hat{n}(\hat{t})$  is an unitary vector which is normal or tangential to the interface of the surfaces, respectively.

As a consequence of j = 0, we have  $\vec{\nabla} \times \vec{H} = \vec{0}$ , implying that

$$\vec{H} = \vec{\nabla}\varphi_m,\tag{10}$$

where  $\varphi_m$  is a scalar potential. At this point, we must remember the definition of the magnetic induction vector

$$\vec{B} = \mu \vec{H} \tag{11}$$

and substituting (10) into (9), it is found that

$$\frac{\partial \varphi_{m_1}}{\partial t} - \frac{\partial \varphi_{m_2}}{\partial t} = 0, \tag{12}$$

where  $\frac{\partial}{\partial t}$  is the tangential derivate, hence

$$\varphi_{m_1} = \varphi_{m_2}.\tag{13}$$

It is important to remark on the assumption that  $\varphi_{m_1}(r) = \varphi_{m_2}(r) = 0$ , as  $r \to \infty$ . So, substituting equations (11) and (10) into equation (8) and using the normal derivate the following equation can be obtained relating the normal component of the magnetic field at each side:

$$\mu_{0}\frac{\partial\varphi_{m_{1}}}{\partial n} = \mu_{1}\frac{\partial\varphi_{m_{1}}}{\partial n}.$$
(14)

The explicit form of  $\varphi_{m_1}$  and  $\varphi_{m_2}$  can now be deduced. This is a point outside the sample which has the magnetic potential  $\phi_m$ ,

$$\varphi_{m_1} = \frac{1}{4\pi} \left[ \frac{m \left( z - d \right)}{r_1^3} + \frac{m_i \left( z + d \right)}{r_2^3} \right],\tag{15}$$

where  $r_1^2 = x^2 + y^2 + (z - d)^2$  and  $r_1^2 = x^2 + y^2 + (z + d)^2$ .

For a point P' located inside the sample, the magnetized medium can be considered as a screening of the original dipole effect, so the magnetic potential will be

$$\varphi_{m_2} = \frac{c}{4\pi} \frac{(z-d)}{r_1^3}, \quad \text{where} \quad c = \text{constant.}$$
(16)

Applying the boundary conditions to  $\varphi_{m_1}$  and  $\varphi_{m_2}$  in the origin, and using equations (10) and (11), we have the following equalities:

$$\mu_{0}\left(\frac{m}{d^{3}} + \frac{m_{i}}{d^{3}}\right) = \mu_{1}\left(\frac{c}{d^{3}}\right) \tag{17}$$

and

$$\frac{m(z-d)}{r_1^3} + \frac{m_i(z+d)}{r_2^3}\Big|_{z=0} = \frac{c(z-d)}{r_1^3}\Big|_{z=0},$$
(18)

and a linear equation system is obtained to determine c and  $m_i$ :

$$\mu_0(m+m_i) = \mu_1 c, \qquad m-m_i = c,$$

where  $\mu_1 = \mu_0 \mu_r$ , with  $\mu_r$  being the relative permittivity. Using the definition  $\mu_1 = \mu_0(1+\chi)$ , we get  $\mu_r = 1 + \chi$ . Hence, the solutions to the equation system are

$$m_i = \frac{\mu_r - 1}{\mu_r + 1}m = \frac{\chi}{\chi + 2}m, \qquad c = \frac{2}{\mu_r + 1}m = \frac{2}{\chi + 2}m.$$

Substituting the first of these equations into equation (7), the following expression is found:

$$F = 3\frac{\mu_0 m^2}{32\pi d} \left(\frac{\chi}{\chi+1}\right). \tag{19}$$

Now, taking  $F = m_0 g$ ,  $\chi$  can be solved to obtain

$$\chi = \frac{2\zeta d^4 F}{m^2 - \zeta d^4 F},\tag{20}$$

where  $\zeta = \frac{32\pi}{3\mu_0} = \frac{8 \times 10^7}{3} \text{A}^2 \text{ N}^{-1}$ .

### 2.2. Model II: solution in a finite medium (cylinder)

Equations (15) and (16) are valid under some important assumptions [9]:

- (i) The sample behaves as a semi-infinite medium.
- (ii) The magnet behaves as a point-like dipole.

Nevertheless, the laboratory conditions do not always fulfil these requirements. In particular, in the case of the determination of  $\chi$  in liquid substances, the sample is a cylinder (finite medium) and the magnet in most of the situations is different from the point-like dipole behaviour. Because of these considerations, the physical model of equations (15) and (16) is not always suitable for application. An improved model, applicable in this case, is the force of one cylinder interacting magnetically with a dipole. The expression of this model has been described in [9] that is

$$F = (\mu_r - 1)\mu_0 \frac{3m^2}{64\pi} \left[ \frac{1}{z_0^4} - \frac{1}{z_1^4} - \frac{z_0^2 + R^2/3}{\left(z_0^2 + R^2\right)^3} + \frac{z_1^2 + R^2/3}{\left(z_1^2 + R^2\right)^3} \right],$$
(21)

here  $\chi = \mu_r - 1$ ,  $z_0$  is the distance from the magnet centre to the bottom of the cylinder (sample),  $z_1$  is the distance from the magnet centre to the top of the liquid column and R is the radius of the cylinder [3]. Equation (21) reduces to equation (19) for a semi-infinite sample, that is, for  $z_1^4 \gg z_0^4$  and  $R^4 \gg \frac{z_0^4}{3}$  as it is expected.

# 3. Experimental methods and results

For diamagnetic and paramagnetic substances the magnetic susceptibility is given by

$$\chi = \left(\frac{64\pi}{3\mu_{\rm o}}\right) \frac{m_{\rm o}gz^4}{m^2},\tag{22}$$



Figure 2. Magnetic measurements fitted to the model of the magnetic moment.

where  $\mu_0, m_0, g, z$  and *m* are the magnetic permeability, the mass—measured in the scale, the gravity acceleration, the distance from dipole's centre to semi-infinite plane modelling the sample, and the magnetic moment, respectively.

For performing the magnetic susceptibility measurements, by using the balance technique, an analytical Mettler Toledo balance, with a precision of 0.01 mg, and a cylindrical magnet of NdFeB with radius r = 0.25 cm and height h = 0.5 cm were used in this study. The magnet was placed over a cylindrical support, which was on the balance plate, as shown in figure 1. Over the magnet, placed as close as it was possible was a glass container (a Petri dish). It is important to emphasize that the Petri dish and the magnet were not in contact.

Once the samples were fixed inside the glass container, the balance was turned on in order to tare it and, according to equation (22), two cases can be observed:

- (i) *Paramagnetic samples.* The magnet has a slight vertical displacement (usually imperceptible for our sight). The fictitious mass recorded in the balance is negative because the magnetic susceptibility  $\chi$  is positive. This is consistent with the case of ferromagnetic samples, in which we can expect a more perceptible motion.
- (ii) *Diamagnetic samples*. The fictitious mass recorded in the balance is positive, implying that  $\chi$  is negative. The motion is, as in the case of paramagnetic samples, quite slight.

#### 3.1. Magnetic characterization

The determination of the magnetic moment was made in two ways. In the first one, the collected data of the magnetic field, measured along the z-axis, were fitted to equation (5), as shown in figure 1. In the second way, the magnetic moment was obtained through the slope of the curve  $dB_z/dz$  versus  $F_z$ , where  $B_z$  is the z component of the magnetic field generated by the small magnet and  $F_z$  is the magnetic force between both magnets, see the fitted data in figure 3(b). In this case, the smallest magnet is used in order to perform this study; it is a disc of radius 1.5 mm and 4 cm thickness. It has a magnetic intensity of 0.37 T in the vicinity of the centre of its face. The measurements of the magnetic field, on both magnets, were repeated

ten times, at 19 different distances (normal to the face of each disc). The magnetic sensor was moved with a space resolution of 1 mm and using a Walker Scientific Inc. gaussmeter, which has a precision of 50  $\mu$ T. The averaged values of the magnetic field were plotted using the software Microcal Origin 6.5 (Massachusetts, USA). The best fitting function was performed taking into account a function with both the dipolar and quadrupolar parameters. Such a function has the general expression

$$B = \frac{1}{z^5} (z^2 m + c), \tag{23}$$

here m and c are the magnetic dipolar and quadrupolar moments, respectively, and z is the distance from the cylindrical face, where B was measured. A difference of two orders of magnitude between m and c was determined, in the case of the smaller magnet. So, it implies that the magnetic field behaves as a dipole on the z-axis (for distances larger than the diameter of the magnet).

Once the magnetic moment of the magnet was measured, the recording of the magnetic moment of our main magnet was performed. A magnetic dipole  $\vec{m}$  inside a non-uniform magnetic field  $\vec{B}$  undergoes an external force  $\vec{F}$ , if it is assumed that  $m_z || B$ , then

$$F_z = m \frac{\mathrm{d}}{\mathrm{d}z} B$$
, due to  $\vec{B} = \hat{k} B$ . (24)

Thus measuring  $F_z$  for several distances from the magnet in the setup, it is possible to estimate the magnetic moment of the larger magnet, in this case it was used to calculate values on equation (19) for every single distance with the parameters *m* and *c* obtained before. The magnetic moment of the main magnet was also estimated with the above procedure, see figures 3(a) and (b).

It is important to emphasize that equation (20) was valid for an ideal magnetic dipole, thus to obtain the correct value of the magnetic moment, using the analytical scale, the separation between the magnets must be greater than their radius. Therefore, the value obtained has a 5% difference according to the estimation carried out through the classic procedure.

#### 3.2. Semi-infinite plane determination

The magnetic lines travelling from the north to the south pole are not straight, they are curved coming from one of the faces of the magnet and going into the opposite face of the cylinder. Therefore, in each circular plane over one face of the magnet, it will be possible to find magnetic field lines travelling along the direction of the magnet axis. Thus, how is the magnetized sample affected? In this case, the magnetization of a physical system is proportional to the geometry of the sample. Our interactive model is based on the determination of the magnetic force between a dipole and a finite or even better an infinite plane, but to obtain a plane which contains all the magnetic field lines is not possible, indeed, in this case is unnecessary because the  $B_x$ ,  $B_y$  dipolar magnetic field components vanish very quickly, it is sufficient to find a 'semi-infinite plane' with  $B_z$  in only one sense, the sense of the magnetic moment of the magnet. So that, the magnetic field was measured at six distances along the cylindrical axis in which the measured substances showed the stronger magnetic interaction with the magnet, but in each selected point, a circular map of  $B_{z}$  was additionally performed to several radii with the purpose of finding the change of direction. In figure 4, the  $B_7$  measurements to several radii is presented. The semi-infinite planes were determined in a range from 1 mm to 6 mm over the cylindrical magnetic face and diameters from 1.3 cm to 3 cm were found according the above criterion.



Figure 3. Magnetic moment of the main magnet using: (a) the balance device and (b) the classic procedure.

#### 3.3. Validation of the measurement modality

In order to perform the validation of the magnetic susceptibility measurement modality, the recordings of five samples weighing 1.00 g [13], were reproduced. Four of the five samples were compressed with a pressure of 6 tons, so that, cylindrical tablets of diameter of  $(18 \pm 2)$  mm and heights of  $(2 \pm 1)$  mm and the fifth sample, a liquid substance held in a small glass bowl with the dimension above specified. The magnetic susceptibility measurements of the five samples are shown in table 2. The first column from left to right indicates the chemical substances; the second column reported the magnetic molar susceptibility  $\chi_M$ , the third column reported the molar mass of the substances, the fourth column reported the density of the substances, the fifth column reported volumetric magnetic susceptibility  $\chi$  measured with the experimental setup proposed by using the model of the infinity in equation (19) and finite plane in equation (21), respectively, the propagated uncertainty and the standard deviations are included. All  $\chi_M$  were transformed for its corresponding  $\chi$  in accordance with table 1.



**Figure 4.** Magnetic field intensity measured as a function of the radial length, for several fixed positions in z, in order to determine the dimensions of the semi-infinite plane.

Table 2. Magnetic susceptibility of 1 g of different chemical substances.

Substance	$\chi_M(\rm cm^3mol^{-1})$	$W_a$ (kg mol <sup>-1</sup> )	$\rho~(\rm kg~m^{-3})$	$\chi   imes  10^{-5}$	$\chi_{inf} \times 10^{-5}$	$\chi_{finite} \times 10^{-5}$
$MnSO_4 \cdot H_2O$	0.0142	0.169	2950	311	$278 \pm 20$	$291 \pm 20$
$MnCl_2\cdot 4H_2O$	0.0146	0.198	2010	186	$214 \pm 20$	$204 \pm 20$
$CuSO_4\cdot 5H_2O$	0.0015	0.250	2280	16.7	$19 \pm 2$	$18 \pm 2$
$NiCl_2 \cdot 6H_2O$	0.00424	0.234	2377	54.1	$51 \pm 3$	$50 \pm 3$
H <sub>2</sub> O	$-1.297\times10^{-5}$	0.018	1000	-0.91	$-0.95\pm0.08$	$-0.94\pm0.08$

#### 3.4. Uncertainty propagation

For magnetic susceptibility measurements, several parameters were recorded as described above, therefore the uncertainty propagation was calculated in the following manner taking into consideration the following parameters:

- (i) distance between the geometric centre of semi-infinite plane and the magnet's geometric centre,
- (ii) the magnetic moment of the main magnet,
- (iii) gravity acceleration and
- (iv) the fictitious mass,

$$\left(\frac{\Delta\chi}{\chi}\right)^2 = 16\left(\frac{\Delta z}{z}\right)^2 + 4\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta m_0}{m_0}\right)^2 + \left(\frac{\Delta g}{g}\right)^2.$$
 (25)

It is important to point out that, the relative uncertainty of the mass and gravity acceleration  $g = 9.8 \text{ m s}^{-2}$ , which are not important due to the relative uncertainty of the magnetic moment m, so this quantity was determined in two different ways and separation distance z was larger, indeed the relative uncertainties in the separation distance were also minimized by determining them with a micrometer.

Moreover, averaging the magnetic moment measured gave  $m = 0.13 \pm 0.05$  A m<sup>2</sup>, and the uncertainty propagation term is

$$4\left(\frac{\Delta m}{m}\right)^2 = 2.4 \times 10^{-4},$$
 (26)

therefore,

$$\left(\frac{\Delta\chi}{\chi}\right)^2 \approx 2.4 \times 10^{-4},\tag{27}$$

and thus the systematic uncertainties in the magnetic susceptibility were  $\Delta \chi \approx 0.02 \chi$ , in other words we have a systematic uncertainty of 2% of the susceptibility.

# 4. Discussion and conclusion

In this work, a description of the methodology and a detailed calibration of one experimental setup for the measurement of the magnetic susceptibility were presented. In order to make a good determination of the best semi-infinite magnetization plane, the magnetic moment of the magnet and the relative uncertainty were necessary. Also a direct comparison between the magnetic susceptibility with a finite and infinite model for the sample was evaluated, showing differences from 5% to 10%.

So, it is considered an important and clear description of the methodology, such that it should be taught and implemented in scientific and engineering experimental physics courses. Also, it should be adopted as a working tool in the chemistry research centers for magnetic sample characterization.

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