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2010 Eur. J. Phys. 31 801

(http://iopscience.iop.org/0143-0807/31/4/009)

IOP PUBLISHING EUROPEAN JOURNAL OF PHYSICS

Eur. J. Phys. **31** (2010) 801–809 doi:10.1088/0143-0807/31/4/009

Lecture-room interference demo using a glass plate and a laser beam focused on it

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Received 17 February 2010, in final form 13 April 2010
Published 26 May 2010
Online at stacks.iop.org/EJP/31/801

Abstract

We describe a simple case of non-localized interference produced with a glass plate and a laser beam focused on it. The proposed setup for observing interference is compact when semiconductor lasers are employed, and it is well suited for demonstration and comparison of interference in reflected and transmitted light in a large lecture-room. This paper is intended for graduate students as well as for those who teach courses in optics accompanied with cost-effective demos.

(Some figures in this article are in colour only in the electronic version)

Introduction

Many interferometers employ a method of amplitude division of a light source through reflections and refractions at the boundaries of media [1]. Long before the advent of lasers, Pohl [2] demonstrated the interference of light to a large audience with the help of a mercury lamp and a thin mica leaf. This excellent experiment still retains its high value. But the opportunities for demonstrating interference were extended, essentially after lasers were created in 1960. Lasers as sources of intense coherent light permitted us to observe bright interference patterns with a large path difference between the beams building the interference. In a number of papers [3–6] the authors used glass plates to observe the interference. Murty [3] used a parallel beam widened with a lens which was incident at the angle $\varphi = 49^\circ$ on a plane-parallel plate. The plate's thickness was in the range $h = 2.5$–5 cm. Interference was observed in the region where the beams reflected by two plate surfaces overlapped, being distorted by aberrations due to the lens. Shustin *et al* [4] proposed to demonstrate the equal inclination bands employing the laser beam diverging after focusing and falling normally on a plane-parallel glass plate of thickness $h = 1.5$ cm. Interference rings are observed on the screen having a small orifice to let the laser beam through and located between the focusing lens and the plate. Formulas describing the interference due to reflection by the plate of two diverging beams created with a slit [5] or a point [6] coherent source were obtained. Rather thick glass plates were used: $h = 8$ mm [5] and $h = 20$ mm [6]. The dependences of the frequency of interference fringes on the incidence angle of the diverging beam onto the plate were also studied there. In all papers noted, a red beam of the He–Ne laser was used. Both Murty [3] and Shustin *et al* [4] noted a high brightness of interference patterns that may be observed in the presence of weak room light. Some typical demo experiments for observing interference with light beams reflected from two sides of a glass plate are also described in a recent book [7].

This paper describes the demonstration of interference to a large audience. We use an ordinary glass plate of moderate thickness ($h = 1$–2 mm) and a laser beam focused directly on the plate. Our setup is intended for lecture experiments and it permits us to demonstrate light interference under interaction with a flat plate in reflected as well as in transmitted light and to study how the incidence angle of the light beam affects both interference patterns. Thus, with its help one can reproduce the experiments with the interference of transmitted light when a diverging beam falls normally on a flat plate [4] as well as the interference of reflected light when a light beam falls on an oblique surface, as described in [7]. In addition, it permits us to show to a large audience the difference in the visibility of both interference patterns for different values of the incidence angle. Again, beam focusing on the plate enables one to perform interference observations for large values of the incidence angle (up to 80°) on plates of moderate size. This paper is intended for graduate students as well as for those who teach courses in optics accompanied with cost-effective demos.

Interference observed due to a focused laser beam reflected by a plane parallel plate

Figure 1 depicts the interferometer setup. A narrow, weakly diverging laser beam passes through a collecting lens (we used a lens with the focal length $f = 9.4$ cm; the lens is not shown in the figure), and the converging beam is focused onto the plate. This figure shows with solid lines the axial ray of the focused beam incident on the plate at an angle φ, and the parallel axial rays 1 and 2 of two diverging beams formed by reflections from the plate surfaces. Broken lines show the internal border rays of these diverging beams.

The plate is put on a small horizontal goniometer device permitting us to vary the incidence angle φ of a focused beam. Two diverging beams reflected by the plate start to overlap already at a moderate distance L_1 from the plate. Overlapping becomes almost total at $L \gg L_1$ and the two beams propagate further as a single diverging beam. A small relative shift of beams is determined by refraction within the plate and its thickness, and at large L it is small compared to the size of beam cross sections. In the transmitted light there are also beams generating two-beam interference. One of them starts from point E, and another forms by reflection at point B.

Let us consider the formation of the interference pattern in the reflected light. It occurs in the total region of beams overlapping, and it may be observed on a screen put in the path of their propagation. On the screen that may be placed at different distances L, we observe a bright light spot with actually straight interference fringes oriented perpendicular to the light incidence plane on the plate and, correspondingly, to the plane where rays of beams 1 and 2 converge. The light spot diameter D and the pattern period Λ increase with growing L. Such a pattern is close to that for two intersecting plane monochromatic waves [1].

The main property of the setup is the application of laser beam focusing increasing the divergence of the beams reflected from the plate, and it permits us to obtain a light spot of

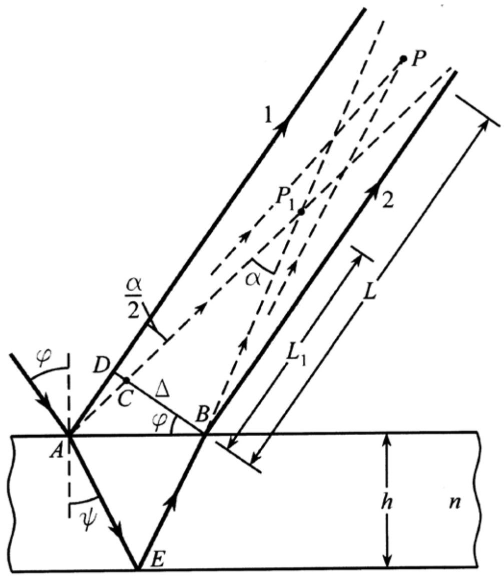


Figure 1. Interference scheme of two diverging beams 1 and 2 reflected from the plate.

rather large size on the screen at a large distance L. At the same time, the angular divergence with scheme parameters noted above does not exceed $\theta = 1^\circ$. As a result, with sufficiently large L, the interference period Λ turns out to be large, and the pattern may be conveniently observed within the light spot remaining bright due to a small θ value. Narrow laser beams determine another important property: at large L, the interfering waves possess wave fronts with small curvature, and the problem of their interference may be solved in the plane wave approximation.

A non-focused laser beam possesses a very small angular divergence ($\theta \approx 10^{-3}$ rad). On employing such a beam, it is almost impossible to obtain the intersection of reflected (and transmitted) beams at an acceptable distance L, the beam diameter D remains small, and in the remote region of their intersection, the interference period Λ is comparable with the diameter D. Besides, conventional glass plates are not strictly plane-parallel. Therefore, narrow, in fact parallel, beams after reflections either would intersect between themselves in a limited space volume or would not interact at all.

Calculating the interference period

In [5], the formula for the frequency N of the interference bands is derived using the model of two imaginary sources created by two sides of the plate on reflecting the light emitted by a slit. Here, we present the derivation of the formula for the interference period $\Lambda = 1/N$ on the ground of the interference model between two plane waves.

We know that under the interference of two plane monochromatic waves with a wavelength λ, converging at a small angle α, the period Λ of the pattern in the plane perpendicular to the bisector of the angle α, is equal to [1]

$$\Lambda = \frac{\lambda}{\alpha}. \tag{1}$$

For calculating Λ with a given value of the laser beam wavelength, one needs to know the angle α. We will use figure 1 for calculating α with some preliminary explanations. We show there the interfering beams 1 and 2 as if they originate at points A and B located at the front surface of the plate. In fact, beam 2 is reflected at point E at the back surface of the plate, and we must prove the opportunity of transferring the beam origin to point B. The first circumstance permitting us to do so is the large length of the beam neck focused with the lens onto the plate. We will make the estimate for the Gaussian beam of a He–Ne laser ($\lambda =$ 633 nm). In this case, the beam radius on the lens was $\omega_s \approx 0.8$ mm, the beam radius after the lens was $\omega_0 \approx 0.012$ mm and the neck length (Rayleigh length) was $2\pi\omega_0^2\lambda^{-1} \approx 1.4$ mm which is comparable with the plate thickness h. Apart from a He–Ne laser we also employed an injection semiconductor laser. Though the beam of a semiconductor laser is not a Gaussian one, the evaluation of the focus length remains approximately valid. (One may observe and even measure the beam pattern near the focus of the lens by removing the plate and using smoke to visualize the whole picture.) In the experiment, the location of the neck relative to the plate surfaces was not fixed accurately. It is sufficient for the plate to be somewhere within the limits of the focus length. The main reason why we can transfer the beam 2 origin along its axis within the plate thickness is determined by the inequality $h \ll L$. While we are interested here in the period Λ but not in the location of interference fringes in the plane of observation, we can use the above formula and disregard the path difference between rays hitting different points of this plane. Evidently, the value of the angle is different for different points, but with $h \ll L$, these variations will be insignificant and will not lead to noticeable Λ variations. It is clear from figure 1 that the angle α, taking into account its small value, is equal to the ratio BC/L. We find the BC length from the triangles ADB and ADC and come to the following formula for the angle α (see the appendix for derivation details):

$$\alpha = \frac{h \sin 2\varphi}{L(n^2 - \sin^2\varphi)^{1/2} + h\sin^2\varphi}. \tag{2}$$

In the approximation $h \ll L$ we obtain the period of the interference pattern:

$$\Lambda = \frac{\lambda L(n^2 - \sin^2\varphi)^{1/2}}{h \sin 2\varphi}. \tag{3}$$

This formula coincides with a similar one obtained in [5].

Experimental results

The application of a plate with a moderate thickness and the beam focusing directly on the plate permit us to observe the interference at large incidence angles up to $\varphi = 80^\circ$ without any difficulty. The plate length in the plane of incidence is not large, being about 4–5 cm.

We proved formula (3) in experiment measuring the relations $\Lambda(\varphi)$ and $\Lambda(L)$ with the He–Ne laser beam and a glass plate with $h = 1.7$ mm and $n = 1.5$. We studied the relation of $\Lambda(\varphi)$ with $L = 150$ cm within the angle range $\varphi = 10^\circ$–80°. In agreement with (3), the registered period of the pattern decreases steadily with growing φ, and then it runs through a spread minimum within the range $\varphi \approx 45^\circ$–60° and then grows at $\varphi > 60^\circ$ (figure 2).

The function $\Lambda(L)$ is studied at $\varphi = 20^\circ$ in the range $L = 25$–200 cm and it shows, in agreement with (3), a linear growth of Λ against L (figure 2). Figure 3 depicts examples of the observed interference patterns ($\varphi = 20^\circ$)

Large brightness of laser beams and linear dependence $\Lambda(L)$ permit us to demonstrate the interference to a large audience. It is important that Λ does not depend on the focal length f of the lens employed for laser beam focusing. Indeed, the angle α (figure 1) for any plane depends only on the BC/L ratio and not on the divergence angle of the beam θ, which depends on f. At the same time, f determines the diameter D of the light spot observed and the number

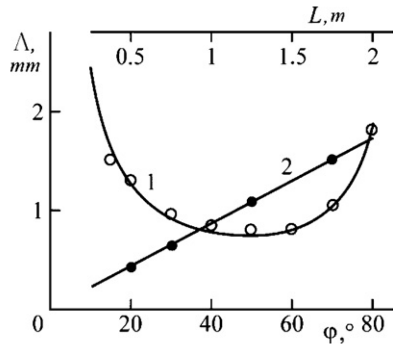
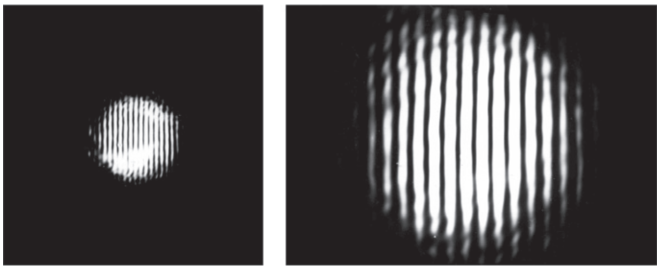


Figure 2. Calculated and registered dependences: $\Lambda(\varphi)$ (1) and $\Lambda(L)$ (2).



(a) (b)

Figure 3. Interference patterns at $L = 0.5$ m (a) and 1.5 m (b). Light source is a He–Ne laser ($\lambda = 633$ nm).

of interference fringes within this spot. For a Gaussian beam the angle $\theta \approx \omega_s/f$ comprised in this experiment is 0.0085 rad. The spot diameter on the screen is $D = 2\theta \cdot L$. Let us take $L = 10$ m. Then we obtain $D \approx 17$ cm, the period of the pattern calculated according to (3) amounts to $\Lambda \approx 1$ cm, and the number of interference fringes in the spot $M = D/\Lambda$ exceeds 15, and in a darkened room such a pattern will be distinctly observed, not only from the noted distance L, but also for a more remote observer. One would observe the pattern without strain. Indeed, the angular size of the pattern period $\Lambda/D = 0.001$ rad is an order of magnitude higher than the diffraction minimum resolved angular distance of a normal eye $1.22\lambda/d_{\text{eye}} = 0.0002$ rad ($d_{\text{eye}} = 4$ mm is the eye pupil diameter).

Interference setup size

The He–Ne laser possesses the radiator length of about 75 cm and furnishes a continuous, linear polarized beam with $P = 10$ mW in power. In experiments, we recommend to employ the polarization of the beam directed across the incidence plane to avoid difficulties associated with the Brewster phenomenon. One may change the direction of polarization through rotating a radiator around its axis. All elements of the setup are fixed rigidly to the optical bench. When the noted laser is employed, the total length of the setup is about 1 m.

Apart from the He–Ne laser, we employed an injection semiconductor laser ($\lambda = 655$ nm, $P \approx 7$ mW, see figure 4).

In the simplest case one may employ a laser pointer with a radiator length about 6 cm. The low cost of pointers enables one to choose one with the highest power possible. However, with a standard supply the power conventionally does not exceed 3 mW. It turns out that

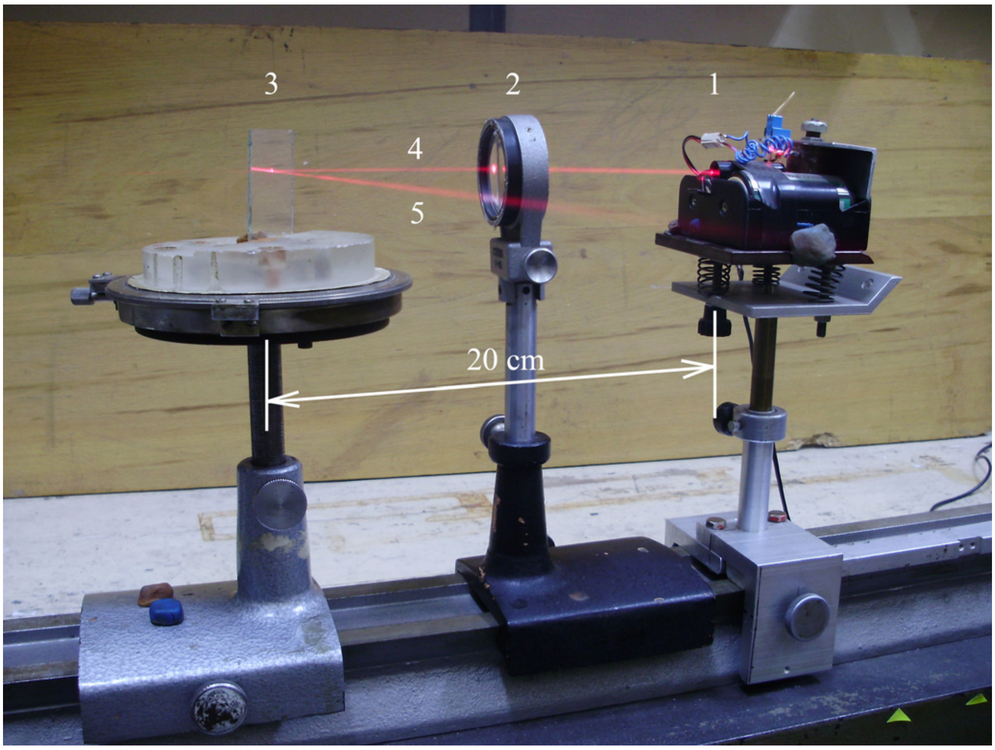


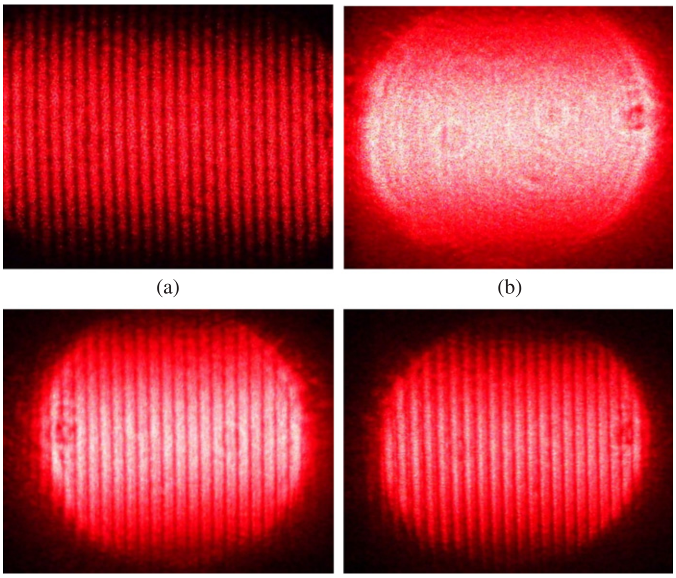
Figure 4. Photograph of the optical setup. 1 is the semiconductor laser together with the supply unit; 2 is the lens focusing the beam on the glass plate, 3 is the glass plate on the goniometer, 4 is the beam incident on the plate, 5 is the beam reflected by the plate.

feeding a laser with AA batteries enhances their radiation power to 5–7 mW, and the laser may operate for many hours. Certainly, one may employ other semiconductor modules supplied with small-size low dc voltage sources fed from the mains that may actually operate for a long time. It is desirable to choose a module with the radiation power $P \approx 10$–20 mW. We note that near the output window the beam of a semiconductor laser possesses a cross section close to an elliptic one. The beam has linear polarization along the small axis of the ellipse. The degree of linear polarization exceeds 90%. Semiconductor lasers are attractive in that they permit us to build a moderate-size interference setup. The distance from the laser output window to a lens may be even several centimetres, and the distance between the laser and the plate may be set not exceeding 20 cm. Moderate size of the setup and fixing elements to the optical bench ensure high stability of the interference pattern against external mechanical vibrations. Moderate-size setups may be conveniently moved for observation with safe orientation of laser beams with respect to spectators.

Comparing the interference in reflected and transmitted light

For the light beam polarized transverse to the incidence plane, the reflection coefficient for the glass–air interface is equal to [1]

$$R_{\perp} = \frac{\sin^2\left\lfloor \varphi - \arcsin\left(\frac{\sin\varphi}{n}\right)\right\rfloor}{\sin^2\left\lfloor \varphi + \arcsin\left(\frac{\sin\varphi}{n}\right)\right\rfloor}. \tag{4}$$



(a) (b) (c) (d)

Figure 5. Interference patterns in reflected (a, c) and transmitted (b, d) light. (a), (b) for $\varphi = 20^\circ$; (c), (d) for $\varphi = 80^\circ$; photographs were obtained with a semiconductor laser ($\lambda = 655$ nm) at $L \approx 4.5$ m from the plate to the screen.

For the normal incidence on the boundary, the reflection coefficient is

$$R = \left(\frac{n-1}{n+1}\right)^2. \tag{5}$$

Let us evaluate the visibility V of the interference pattern in the reflected and transmitted light. We will regard the laser radiation as totally coherent. Under the two-beam interference the visibility is [1]

$$V = \frac{2\sqrt{I_1 \bullet I_2}}{I_1 + I_2}, \tag{6}$$

where I_1 and I_2 are the intensities of the first and second beam, respectively, and related to the intensity I_0 of the beam incident on the plate.

When the incident angle is $\varphi \leqslant 20^\circ$, then we have $R_\perp \approx R = 0.04$. In the light reflected from the plate we obtain for the intensity of the first beam reflected from the front boundary the formula $I_1 = RI_0 = 0.04I_0$. For the second beam reflected from the back boundary, the intensity is $I_1 = (1-R)^2 RI_0 = 0.037I_0$ and from (3) we obtain $V \approx 1$.

For the transmitted light the first beam crosses the plate without reflection inside it: $I_1 = (1-R)^2 I_0 = 0.92I_0$. The second beam experiences two reflections inside the plate from its boundaries: $I_2 = (1-R)^2 R^2 I_0 = 0.0015I_0$. Visibility equals $V = 0.08$ and in fact, the interference pattern in the transmitted light is not observed.

For $\varphi = 80^\circ$ and the same two beams we obtain from (4) and (6): $I_1 = 0.54I_0$, $I_2 = 0.11I_0$ and $V = 0.75$ in reflected light; $I_1 = 0.21I_0$, $I_2 = 0.06I_0$ and $V = 0.83$ in transmitted light. Thus, at large angle of incidence, the two-beam interference possesses high and approximately equal visibilities in reflected as well as in transmitted light.

The calculated results obtained are illustrated with registered patterns shown in figure 5.

The patterns were obtained with the semiconductor laser in reflected and transmitted light with the distance to the screen $L \approx 4.5$ m. In order to observe two patterns at close screens or a single large screen at small φ simultaneously, the reflected beam may be diverted into the required direction with a mirror with an outer reflecting coating. The non-uniformities (Airy patterns) observed on some photographs of figure 5 are due to dust particles deposited on the surfaces of the optical setup, and they make no obstacle for observing interference in a large room.

Conclusions

We described a simple case of non-localized interference achieved with a conventional glass plate and a laser beam focused on it. The proposed setup for observing interference is compact when semiconductor lasers are employed, and it is well suited for demonstration and comparison of interference in reflected and transmitted light in a large lecture-room.

Appendix

Derivation of the formula for α (2)

From figure 1 and triangles ADB and ADC we have

$$BC = BD - CD \tag{A.1}$$

$$BD = AB\cos\varphi \tag{A.2}$$

$$AB = 2h\tan\psi. \tag{A.3}$$

Inserting (A.3) into (A.2) we obtain

$$BD = 2h\cos\varphi\ \tan\psi \tag{A.4}$$

$$CD = AD\tan(\alpha/2) \tag{A.5}$$

$$AD = AB\sin\varphi. \tag{A.6}$$

Inserting all above results into (A.1) we obtain

$$BC = 2h\cos\varphi\tan\psi - 2h\tan\psi\tan(\alpha/2)\sin\varphi. \tag{A.7}$$

Let us calculate $\alpha = \frac{BC}{L}$ using the above results and approximating the tan function with its argument:

$$\alpha = \frac{2h\cos\varphi\tan\psi - \alpha h\tan\psi\sin\varphi}{L}. \tag{A.8}$$

Now we find for α the following formula:

$$\alpha = \frac{2h\cos\varphi\tan\psi}{L + h\tan\psi\sin\varphi}.$$

Let us make the transformation, using Snell's law $\frac{\sin\varphi}{\sin\psi} = n$:

$$\tan\psi = \frac{\sin\psi}{\cos\psi} = \frac{\sin\varphi}{\sin\varphi}\frac{\sin\varphi}{\cos\psi} = \frac{\sin\varphi}{n\cos\psi}.$$

$$\cos\psi = \sqrt{1-\sin^2\psi} = \sqrt{1-\frac{\sin^2\varphi}{n^2}} = \frac{\sqrt{n^2-\sin^2\varphi}}{n}.$$

Then the formula for α assumes the form

$$\alpha = \frac{h\sin^2\varphi}{L\sqrt{n^2-\sin^2\varphi + h\sin^2\varphi}}. \tag{A.9}$$

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