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# The buzzer-a novel physical perspective on a classical toy 

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#### Abstract

We present a physically interesting toy, which is easily constructed and operated-the so-called buzzer. In spite of its simplicity, its physical analysis turns out to be rather complex. Thus, it comes as no surprise that most of its users are not familiar with the underlying physical mechanism. In this paper we propose a physical model which allows for the qualitative and quantitative description of the fundamental physical properties of the buzzer and report on the good agreement between theoretical and experimental data. The model is designed to give a basis for further simplification.


## 1. Introduction

Although toys are not physical objects per se, many of them may play an important role in the teaching of physics. Unlike physical instruments and apparatus, which are exclusively made to demonstrate physical laws, toys like other objects of the everyday-life world do not contain the physical aspects for direct reading. The physical aspects have to be wrested from them and require creative activities which are in some respects similar to real physical research [1]. This may be one reason why many famous physicists had been attracted by physical toys and toying [2]. Although toys are familiar, simple in operation and sometimes even in construction, the physical description can be rather challenging. In many cases the motivation for a physical engagement of the students comes to an early end because the physical description of even simple looking toys turns out to be too complicated. Therefore, physical approaches of at least the well-known and classical toys should be available.

The main purpose of this paper is to present a physical description of a classical and simple looking toy which-as far as we know-is lacking until now. This toy has many names amongst which 'buzzer' is probably the most general. It is also known as 'buzz', 'whizzer', 'magic wheel', 'saw-mill', 'bullroarer' or simply 'button-on-a-string'. The buzzer is easily constructed in a few minutes and consists of an object that usually has two holes on each side of its centre of gravity. Through these a cord is fed, one at a time. The ends of the cord are


Figure 1. Already in the Middle Ages 'buzz bones' were very popular as a toy (drawing: Ewa Rehwald).


Figure 2. The buzzer can be constructed very quickly from a cord loop and a button, as has been done since ages by the children.
then tied together in a way which allows for a finger to be put through the loops on both sides (figure 1).

The cord has then to be wound up by a whirling motion of one's hands. If a tension is applied to the ends of the hence twisted cord, it will unwind and, depending on the quantity of the inertial moment of the rotation, rewind in the opposite direction. By applying and releasing tension in an appropriate rhythm, this process, which is accompanied by the typical buzzing or humming sound, can be repeated indefinitely. According to the Hornbostel-Sachs scheme of musical instrument classification [4] and in view of its eponymous acoustic properties, this extraordinary toy could also be classified as a 'free aerophone', as well.

It is however not necessary to use a medieval 'buzz bone' as a rotator to achieve the intended effect. A big button, as the one shown in figure 2, is more suitable, anyway, for it is well balanced and already equipped with the necessary holes.

In order to give a general physical approach of the buzzer, the level of readership is designed for graduate students and general physicists and could serve as a basis for further simplification for lower levels of physical instruction.


Figure 3. Experimental setup: (a) buzzer with a chopper wheel, (b) driving motor, (c) tilting lever with return spring, (d) dynamometer, (e) light barrier and (f) travel sensor.

Despite its simple makeup, the quantitative analysis of the buzzer's underlying physical mechanism turns out to be very complex. Unsurprisingly, a physical model appropriate for high-school and college physics does not exist. The main objective of our investigations was thus the development and experimental verification of a quantitative model. Albeit its innately mathematical nature, the latter may serve as a basis for a more general, qualitative analysis, as well.

## 2. The buzzer on the rack

The remarkable behaviour of a buzzer largely depends on its operator's rhythmical application of tension to its cord. Experienced 'players' know implicitly that one may control the revolutions per cycle and the angular velocity of the rotator by appropriately matching the evolution of the tractive force to the pulling frequency.

In order to study the toy's behaviour independently of individual influences, it was yet necessary to construct a device which applies and releases the tension in a reproducible and easily quantifiable way. For this purpose, one end of the buzzer's cord was tightly connected to a dynamometer, while the other end was fixed to a tilting lever, which could be moved back and forth periodically by means of a motorized driving mechanism. The resultant linear displacement was registered by a travel sensor. The rotator, itself, was attached to a light chopper wheel, whose row of slots interrupted the beam of a light barrier, so that the angle of rotation and the angular velocity could be determined based on the measured number and frequency of interruptions. The experimental setup shown in figure 3 needed an initial transient


Figure 4. The angular velocity $\dot{\varphi}$ and angle of rotation $\varphi$ versus time.
period of about 10 s after which the 'amplitude' of the angle of rotation $\varphi$ became steady and the oscillation period depended exclusively on the constant excitation frequency of the driving device.

The time behaviour of the angle of rotation $\varphi$ and the angular velocity $\dot{\varphi}$ of the buzzer disc at steady state is shown in figure 4 . In this case the inertial moment of the buzzer disc was $I=2.48 \times 10^{-5} \mathrm{Nms}$ and the maximum span length of the cord amounted to 70 cm . With these parameters the buzzer could be operated with excitation frequencies between 0.18 Hz and 0.25 Hz .

The time series is at least in principle infinite. In order to get a more compact representation of the system's final behaviour (after transients have vanished), it is convenient to plot the variables of the system as coordinates in the state space (also called phase state) [3]. A very simple example is the phase state of a harmonic oscillation where the angular velocity is plotted against the angle. In this case the trajectory is just a circle. For the buzzer the angular velocity $\dot{\varphi}(t)$, the tractive force $F(t)$ applied to the cord and the distance $z(t)$ travelled in the course of its elongation are used as coordinates. The respective vector triple traces a characteristic orbit. Three projections of an orbit obtained from the measured data are shown in figure 5. About four cycles have been plotted showing that the behaviour of the buzzer is nearly exactly repeated in each cycle. Only minor deviations indicate that a real system is always exposed to random perturbations.

The buzzer may be perceived as a dissipative system: the energy input from outside is dissipated by friction within the system. The area framed by the projection of the orbit $F(z)$ corresponds to the energy transferred into the system, whereas the kinetic energy contained in the system may be inferred from the corresponding values of $\dot{\varphi}$. The kinetic energy vanishes if $\dot{\varphi}=0$, i.e. when the absolute value of the angle of rotation cycles its maximum.

## 3. A physical model of the buzzer

By adopting some simplifying assumptions, a rather simple equation of motion may be derived. In order to reduce the complexity, we initially neglect the weight of the buzzer. This parallels the assumption that it was driven in zero gravity. In this case, the axis of rotation coincides


Figure 5. State space spanned by $\dot{\varphi}, F$ and $z$. The trajectory determined by the measured data (not plotted here) has been projected on three planes.


Figure 6. The trajectory of the variation of the angle of rotation against the displacement (measured values). Again about four cycles have been plotted.
with the line of action of the tractive forces (figure 7). Furthermore, it is assumed that, in the untwisted state, each of the two cords may be represented by a thin cylinder with a constant cross section. Finally, both the cord's resistance to being twisted and its elasticity are preliminarily neglected, as well.

The total length of the cord amounts to $4 l$, so that the pair of cords has a maximum span length of about $2 l$. The radius of the circular cross section of a single cord is $r$. We further assume that the tractive force $F$ causes a tension which is uniformly distributed across the entire cross section of each cord, so that $F$ attaches at its geometric centre. For simplicity, we further assume that the total tractive force is transmitted by the central filament of each cord and that the material surrounding this filament functions as a simple filler. Consequently, a


Figure 7. If the weight of the buzzer disc is neglected, the axis of rotation will coincide with the line of action of the tractive force.


Figure 8. Assuming that the force is transmitted exclusively by the central filament, the latter spirals on a cylinder with a diameter $2 r$.
twisting of the pair of cords corresponds to a spiral movement of the filament on a cylinder with radius $r$ as it is illustrated on the right-hand side of figure 8 .

By means of this simplified representation it is possible to give a straightforward quantitative description of the torque exerted on the buzzer. By twisting the pair of cords, each segment of the central filament is distorted with respect to the axis of rotation by the helix angle $\vartheta$ shown in figure 9. If, now, a couple of forces $\pm F$ act on the pair of cords in the direction of the axis of rotation, each segment of the central filament will be balanced by the respective forces $\pm F_{s}$.

The orientation of $\pm F_{s}$ deviates from the direction of the axis of rotation by the helix angle $\vartheta$. The component $F_{t}$ orthogonal to the axis is directed tangentially to the surface of the cylinder around which the central filament is wound. Thus, at the upper end of the coil, the pair of filaments must be held in a twisted state by a couple of tangential forces $\pm F_{t}$


Figure 9. Equilibrium of forces at a segment of the force transmitting filament.


Figure 10. The direction in which the twisted part of the twin cord is unwrapped coincides with the line of action of the tractive force $F_{s}$.
(figure 10), which corresponds to a torque of $2 r F_{t}$, where $r$ denotes the distance of the tangential forces from the axis of rotation.

As can be derived directly from the geometric relations in figure $10, F_{t}=F_{a} \tan (\vartheta)$. The component $F_{a}$ being parallel to the axis of rotation transmits half of the axial component of the tractive force per filament. With the axial force being $F_{a}=F / 2$, the tangential force may be expressed by $F_{t}=F \tan (\vartheta) / 2$. The insertion of this result into the previously derived term for the torque delivers $r F \tan (\vartheta)$. If one takes into account that a torque is exerted in the same direction by identical coils on both sides of the disc, the total torque equals

$$
\begin{equation*}
M=2 r F \tan (\vartheta) \tag{1}
\end{equation*}
$$



Figure 11. An inclined plane wrapped around a cylinder serves as a model for the formation of the helix of a filament within the twisted section of a pair of cords. With each rotation of the cylinder the rotation angle $\varphi$ increases by $2 \pi$. At the same time, the side opposite to the helix angle $\vartheta$ increases by $2 \pi r$.

Obviously, the torque depends on the radius $r$ of the cross section of the cord, the tractive force $F$ and the degree of twisting expressed by the helix angle $\vartheta$.

With the last term in (1) being proportional to the degree of twisting and thus the number of windings, $\tan \vartheta$ from equation (1) may be expressed as a function of the angle of rotation $\varphi$ (in unit radians). To illustrate the geometrical relations of these quantities, we consider the spiral of a filament as a spatial curve, which arises from wrapping a rectangular triangle (made out of paper) around a cylindrical pencil (figure 11). The hypotenuse of the triangle would then have the length of the twisted part of the cord.

In the geometry of the buzzer, as is shown in figure 10, the distance from the buzzer's disc to the right end of the twisted cord $k$ constitutes one leg of the rectangular triangle with a hypotenuse of length $l$ and a second leg of length $p$. The distance from the centre of the buzzer's disc to the drill hole through which the cord is fed is $q$.

The unwrapping of the helix delivers an inclined plane, as the one shown in figure 11. The distance $r \varphi$ may be found similarly by unwrapping the twisted part of the twin cord in figure 10. In order to account for the changing sign of $\varphi$, the length of $p$ must be determined from

$$
\begin{equation*}
p=\operatorname{sign}(\varphi) q+r \varphi \tag{2}
\end{equation*}
$$

With $p$ being the side opposite to the helix angle $\vartheta$ and $k$ being the adjacent side (figure 10), we derive the relation

$$
\begin{equation*}
\tan \vartheta=p / k \tag{3}
\end{equation*}
$$

where $k$ may be determined by the means of the Pythagorean theorem: $k=\sqrt{l^{2}-p^{2}}$.
With (2) and (3) all quantities which are necessary to determine the torque exerted on the disc of the buzzer have been found. When the buzzer is operated, however, different frictional forces give rise to an additional torque $M_{R}$, which may be taken into account by assuming $M_{R} \propto \dot{\varphi}^{2}$.

If the inertial moment of the disc about the axis of rotation is denoted by $I$, the equation of motion reads $I \ddot{\varphi}=-M-M_{R}$, or by substituting equation (1),

$$
\begin{equation*}
I \ddot{\varphi}=-2 r F \tan \vartheta-M_{R} \tag{4}
\end{equation*}
$$

After the insertion of the corresponding terms from equations (2)-(4), it finally adopts the explicit form

$$
\begin{equation*}
I \ddot{\varphi}=2 r F \frac{\operatorname{sign}(\varphi) q+r \varphi}{\sqrt{l^{2}-(\operatorname{sign}(\varphi) q+r \varphi)^{2}}} \tag{5}
\end{equation*}
$$



Figure 12. Relation between angle of rotation $\varphi$ and displacement $z$ (model calculation for a non-flexible cord).

In order to calculate the behaviour of a specific buzzer, we still have to determine the time dependence of the tractive force $F(t)$. Similar to a crank and slider mechanism, the rotational motion of the crank of the driving motor is converted into a linear motion. Since both the horizontal and vertical displacements of the crank contribute to the linear motion, the increase of the tractive force at the beginning and the decrease at the end of a cycle are much steeper than it would be in the case of a sinusoidal motion. The assumption

$$
\begin{equation*}
F(t)=A \cdot|\sin (\omega t+\tau)|^{3}+F_{\min } \tag{6}
\end{equation*}
$$

accounts very well for this special behaviour and delivers experimentally verifiable results. Thus, the tractive force $F(t)$ has a minimal value of $F_{\text {min }}$ and it oscillates periodically with amplitude $A$ and a period of $T=2 \pi / \omega$.

## 4. Results of the model calculation

Figure 12 shows the orbits of the calculated relation between the angle of rotation $\varphi$ and the traction distance $z$.

In our calculations the maximum of the traction distance coincides with the maximum of the span length of the cord when $\varphi=0$. Accordingly, at the minimal value of the traction distance, the angle $\varphi$ cycles its maximal value. The difference between the maximal and the minimal length of the cord is about 3 cm (see figure 12).

Due to our simplification by restricting the calculations to a non-flexible cord (figure 12), we get a slight deviation from the experimental measurements figure 6 .

After the implementation of the experimentally investigated parameters into our model, the simulated behaviour of the buzzer (figure 13) can be compared with the real behaviour as shown in figure 5 . In both cases the orbit within the state space spanned by the coordinates $\dot{\varphi}, F$ and $z$ has been plotted after the initial transients have vanished. First of all, it arises that the orbits have the same topology and show roughly the same behaviour. The recognizable differences between figures 5 and 13 are because of the simplification made in simulation that the cord is not flexible. Therefore, the force $F$ develops more uniformly during the displacement $z$ which leads to a more rounded, nearly circular form of the projected orbit


Figure 13. Space of states spanned by $\dot{\varphi}, F$ and $z$. The orbit projected on the three planes has been determined by the model calculation for the buzzer, whose experimentally measured orbit is shown in figure 5 .
on the $F-z$ plane. For the same reason in the $\dot{\varphi}-z$ plane it can be seen that the rotation stops ( $\dot{\varphi}=0$ ) when the displacement $z$ is minimal, i.e. the cord is maximally drilled (figure 13). In contrast, in the real case the disc rotates further when $z$ is minimal (which only can happen, if the elastic cord is stretched) and does not stop before the displacement of the cord increases.

The accordance between the experiment and model could be improved by including the elasticity. But then the model would become so complicated that the gain of accordance is too disproportionate to our intention to make the description at the same time as simple and as correct as possible.

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