in the funnel, and the spring is made to oscillate. The period of oscillation is determined. The funnel is now made to come to rest in the equilibrium position, and the lower of the two 5-in. rings, known as the stopping ring, is brought in contact with the stopping rod.

Now the spring is extended downward, and the funnel fixed in its lower position. When released, the funnel will rise to its equilibrium position where it is stopped. The steel sphere will continue rising to a height determined by the velocity the funnel had when stopped, and the value of the acceleration due to gravity.

The upper of the 5-in. support rings, known as the measuring ring, is adjusted to determine the height to which the sphere rose. This is done by sighting across the top of the ring and making it level with the top of the sphere in its extreme upper position.

The funnel is now held securely against the stopping ring, which places it in its equilibrium position, and a meter stick inserted down the spring and made to rest on the top of the sphere. The height to which the sphere rose can now be read by sighting across the measuring ring. The funnel is now fixed in its lower position, and the height to the measuring ring is taken. The difference between this height and the height to which the sphere rose from the equilibrium position is the distance the funnel and sphere moved before being stopped, and is the measure of the radius of reference circle. This can be used to calculate the initial velocity of the steel sphere.

It is necessary to correct for the damping effect that takes place during the first one-fourth of an oscillation of the spring. This damping effect is obvious when the funnel is allowed to execute a complete vibration. The value of the damping effect for the first one-fourth vibration can be found by allowing the funnel to execute a complete vibration, releasing it from its fixed lower position. Measure the distance by which it fails to return to its position of release. One-fourth of this may then be subtracted from the original value of the radius of the reference circle to obtain the effective radius of the reference circle.

Sample data taken from a run with a $\frac{3}{4}$-in. steel ball in the funnel and 21 turns of spring are:

Time of oscillation of the spring	1.200 sec
Height to which the sphere rose	18.7 cm
Distance from sphere at release until stopped	37.0 cm
One-fourth of damping during first oscillation	0.5 cm
Effective radius of reference circle	36.5 cm

This yields a value of 975 cm/sec^2 for the value of gravitational acceleration.

The student in this experiment is trying to determine a value that is known with a high degree of accuracy. The role of experimental error soon becomes apparent to him. The time of oscillation, in particular, should be the result of averaging several observations so as to obtain four places in the value for time. Accuracy is improved by averaging values obtained for the height to which the ball rose. A meter stick supported so as to pass down the center of the spring to the approximate height to which the ball is expected to rise can improve the values obtained for this observation. More than one-half of the values computed should lie between the values of 975 cm/sec^2 and 985 cm/sec^2.

Demonstration Gyroscope

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RECENTLY, Dosso and Vidal[1] have described large-scale apparatus for angular motion demonstrations, including a bicycle-wheel gyroscope. Leybold (Cat. No. 348 18) produce a large Maxwell top of bicycle-wheel dimensions which can be used for other demonstrations concerning angular motion.

The present note describes an apparatus (Fig. 1) that enables a variety of phenomena involving angular-momentum conservation to be demonstrated to large classes. It consists of a bicycle wheel mounted on the end of an arm pivoted on ball-races to rotate about either a vertical or horizontal axis: for rigidity, the arm is constructed of 1-in. aluminum tubing. An adjustable counter-weight on the arm allows a selected torque to be applied about an axis perpendicular to the angular-momentum axis of the wheel, or for the wheel to spin under the action of zero torque. The rim of the bicycle wheel is loaded with copper wire to increase its moment of inertia. The arm on which the wheel is mounted represents the direction of the angular-momentum vector, and an arrowhead can be clipped on either end of the arm to indicate the sense of

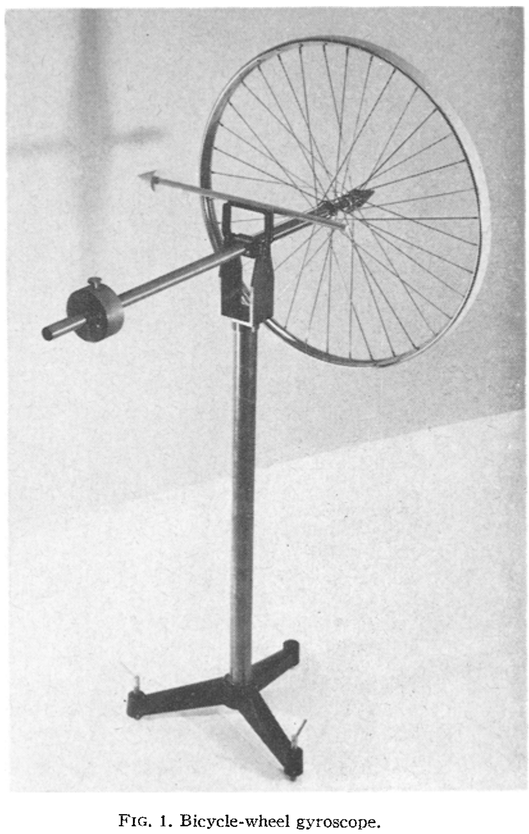

FIG. 1. Bicycle-wheel gyroscope.

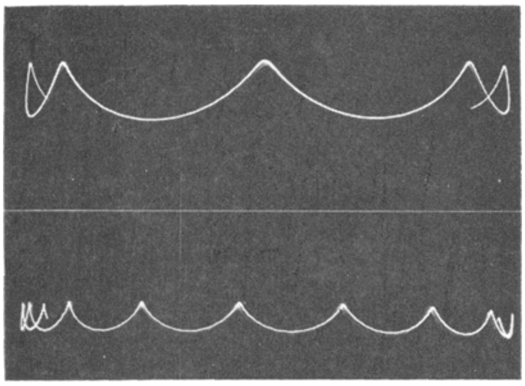

FIG. 2. Cycloidal paths traced by the axis of a gyroscope undergoing nutation.

this vector. A further clip-on arrow shows the direction of the torque vector.

In the absence of external torques, conservation of angular momentum requires that not only should the *magnitude* remain constant, but that the *direction* of the vector should remain fixed in space. If the counterweight is adjusted for zero torque, and the axis set to some fixed direction (for best effect, inclined to the horizontal), the whole apparatus may be carried about the lecture room while the axis continues to point in its original direction. While it is, of course, true that any rigid body, spinning or not, retains its orientation in space in the absence of an external torque, the present demonstration underlines the vector character of angular momentum.

The standard expression $\boldsymbol{\tau}=\boldsymbol{\Omega}\times\mathbf{L}$ relating the torque $\boldsymbol{\tau}$, angular momentum $\mathbf{L}$, and the angular velocity of precession $\boldsymbol{\Omega}$ is illustrated semiquantitatively: Selected torques, preferably in integral ratios, are applied and the precessional-angular velocity measured for a fixed angular momentum. If a strobo-flash unit is available for finding the angular velocity of the wheel, the precessional-angular velocity can be found as a function of angular momentum with a fixed torque. The large size of the apparatus makes evident the relations between the directions of the vectors concerned. It is also easily demonstrated that "hurrying" the precession produces a torque counter to the applied torque.

It is not always appreciated by students that the expression $\boldsymbol{\tau}=\boldsymbol{\Omega}\times\mathbf{L}$ strictly applies only after steady precession is established. If the torque is suddenly applied, the system undergoes nutation. In the present apparatus, the period of nutation is slow enough and its amplitude large enough to be easily seen, and the cycloidal path traced by the end of the angular momentum axis is beautifully exhibited. Figure 2 shows photographs of such cycloids taken with a small lamp fixed to the end of the axis. If the nutation is damped out by means of pads held against the horizontal axis of the apparatus, it settles down with the angular-momentum axis displaced in the sense of the applied torque, demonstrating that a component of the spin angular momentum has been applied to provide the necessary angular momentum about the precession axis.

The prototype of this apparatus was constructed by E. Wood of the University of British Columbia and the present model by J. Steeples of the University of Alberta, Calgary.

[1] H. W. Dosso and R. H. Vidal, Am. J. Phys. **30**, 528 (1962).

Falling Satellite

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FUTURE generations might determine the advent of the space age by plotting, chronologically, the fraction of introductory texts that include a discussion of earth satellites. The introduction of such satellite problems is generally advanced as an application of Newton's law of gravitation. One observes that the satellite experiences a single external force

$$F=GM_em/R^2, \tag{1}$$

where G is the gravitational constant, M_e and m the masses of the earth and satellite, and R the (circular) orbit radius. Using Newton's second law enables one to recognize the resulting "centripetal" acceleration

$$a=v^2/R \quad (v=\text{orbital speed}) \tag{2}$$

as GM_e/R^2. This in turn leads to an expression for the satellite period

$$T=2\pi R(R/GM_e)^{\frac{1}{2}}, \tag{3}$$

which may be compared with the observed periods of nearly 90 min.

If the discussion is dropped at this point, the student may be only partially convinced, being left with the unanswered question, "If the acceleration is gravitational, and radially inward, why doesn't the satellite fall to earth like any other law-abiding body?" The following derivation may prove useful in demonstrating that the orbiting satellite is indeed falling like any gravitational law-abiding body should. In particular, it obeys the equation governing the motion of a freely falling body undergoing a constant acceleration, viz.

$$s=\tfrac{1}{2}at^2, \tag{4}$$

a fact which ought to convince the student that satellites *do* fall.

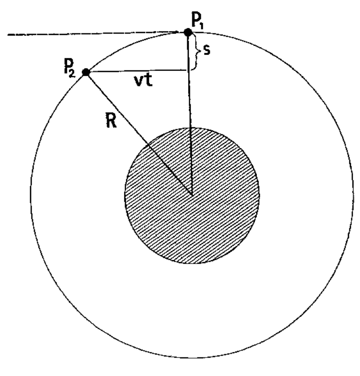

FIG. 1. Falling satellite.