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A laboratory experiment on coupled non-identical pendulums

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Abstract

In this paper, coupled pendulums with different lengths are studied. Through steel magnets, each pendulum is coupled with others, and a stepping motor is used to drive the whole system. To record the data automatically, we designed a data acquisition system with a CCD camera connected to a computer. The coupled system shows in-phase, locked-phase and anti-phase synchronizations when the driving frequency and the coupling strength are changed. With background knowledge from general physics and the simplicity of the equipment, this experiment is easy to implement and would be of interest to undergraduate students.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In 1657, Christian Huygens first observed synchronization of two identical clocks hanging from a common wood beam, and ascribed this phenomenon to the slight motion of the beam [1]. Since then, coupled pendulums have aroused great interest from multi-disciplinary fields (see, e.g., [1–7]). For these studies, many methods of coupling have been designed, such as a beam with pulleys [8, 9] and connecting adjacent pendulums with a torsional spring [10]. Nevertheless, as far as we know, most previous experiments on coupled pendulums could not provide a convenient method for changing the coupling strength.

In our experiment, therefore, we design precise equipment for varying the coupling strength using round steel magnets. We can increase the coupling strength easily by increasing the number of magnets. In addition, different from previous works, we can also study coupled non-identical pendulums with different lengths. We find that non-identical pendulums with different natural frequencies exhibit some interesting phenomena.

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These findings are of great interest not only for researchers in nonlinear dynamics, but also undergraduate students studying general physics. With background knowledge from general physics and the simplicity of the equipment, this experiment can be incorporated into undergraduate laboratories for mechanics or modern experiments. The experiment can provide undergraduate students with a direct and vivid approach to the study of dynamical behaviours of coupled systems.

The rest of the paper is arranged as follows. In section 2, we introduce the experimental setup. In section 3, we present the experimental results and analysis. Finally, we give our conclusions in section 4.

2. Experimental setup

The whole experimental setup is shown schematically in figure 1. Triangle brackets and a stainless bar (non-magnetic conductive material; diameter 8 mm) support the whole system. A number of steel rings are placed on the bar. These rings cannot move along the bar, but can rotate around it. From each ring is suspended a steel ball (its mass and diameter are 33 g and 20 mm, respectively) through a soft cycloid of negligible mass.

A stepping motor (MT42STH47-1684A) is hard connected with one end of the bar. With a controller, shown in figure 2(a), the stepping motor can drive the rotation of the bar left and right on its axis. In our experiment, the bar can rotate between $-60^\circ$ and $60^\circ$. Then, the steel rings are driven by the bar friction. Finally, the pendulums are driven to swing left and right.

The motions of the pendulums are simultaneously captured by a CCD camera placed to the left of the setup. The pendulum balls are decorated with different colours distinct from...
the background to allow the image processing software to recognize them and calculate their displacement. Horizontal displacements \(s\) of all pendulum balls are recorded at the same time, every other 0.04 s. We can then get the angular deflection of the pendulum balls via the formulation \(\theta = \arcsin(s/l)\). With the recorded data, the computer connected to the CCD camera shows the video and traces of pendulums, as shown in figures 2(b) and (c).

To study the effect of coupling, we introduce coupling by putting the same number of round steel magnets on each steel ring, as shown in figure 2(d). Steel magnets on adjacent pendulums attract each other, and their distance is fixed to about 35 mm. Due to the attraction, once there is an angular deviant between magnets which initially face each other, a restoring force is generated to force magnets to return to the face-to-face state. In this way, the swing of one pendulum can cause its adjacent pendulums to swing.

In our experiment, we can adjust coupling strength by changing the number of steel magnets. The coupling strength increases with the number of steel magnets. For example, in figure 1, there is one pair of magnets on every steel ring, but in figure 2(d), there are three pairs of magnets on every steel ring. The magnetic flux density in the middle of the neighbouring magnets is about 30 mT (more data are shown in table 1). So, the more magnets on every steel ring, the stronger the coupling strength is. In our experiment, we measure the magnetic field using a Teslameter (Leybold, 51662).

3. Experimental results and analysis

3.1. Research on two coupled pendulums

3.1.1. Changing driving frequency. First, we investigate the motion of two coupled pendulums. Our stepping motor controller can adjust driving frequency from 3.5 to
Figure 3. Lissajous curves of two coupled pendulums with different driving frequencies in (a)–(e) and the phase difference–frequency characteristic (PFC) curve in (f). \( \theta_1 \) and \( \theta_2 \) represents angular deflections of pendulums \((l_1 = 0.275 \text{ m} \) and \( l_2 = 0.245 \text{ m})\). The circles and dots represent initial unstable and final stable states respectively. (a) \( \omega_1 = 3.79 \text{ rad s}^{-1} \), (b) \( \omega_1 = 5.88 \text{ rad s}^{-1} \), (c) \( \omega_1 = 5.98 \text{ rad s}^{-1} \), (d) \( \omega_1 = 6.05 \text{ rad s}^{-1} \), (e) \( \omega_1 = 7.94 \text{ rad s}^{-1} \) and (f) PFC curve.

Table 1. Magnetic flux density of different pairs of magnets.

<table>
<thead>
<tr>
<th>Number of magnet pairs on every steel ring</th>
<th>Magnetic flux density ( B ) (10(^{-3}) T)</th>
<th>No. 1</th>
<th>No. 2</th>
<th>No. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>36.2</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>27.5</td>
<td>–30.7</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>37.0</td>
<td>–26.8</td>
<td>32.5</td>
</tr>
</tbody>
</table>

8.5 rad s\(^{-1}\) in steps of about 0.04 rad s\(^{-1}\). Then we investigate the synchronous states of two coupled pendulums \((l_1 = 0.275 \text{ m}, \omega_1 = 5.97 \text{ rad s}^{-1}, l_2 = 0.245 \text{ m} \) and \( \omega_2 = 6.32 \text{ rad s}^{-1} \)) with two pairs of coupling magnets. Experimental results are shown in figure 3, where \( \theta_1 \)
and $\theta_2$ represent angular deflections of two coupled pendulums and $\Delta \Phi$ represents the phase difference between two pendulums.

With a sufficiently low driving frequency ($\omega = 3.79$ rad s$^{-1} \ll \omega_1, \omega_2$), we can see that two pendulums reach in-phase synchronization because the relation between $\theta_1$ and $\theta_2$ is depicted as a line of positive slope as shown in figure 3(a). When driving frequency is just a little lower than $\omega_1$ ($\omega = 5.88$ rad s$^{-1}$), pendulums transform into locked-phase synchronization since the Lissajous curve turns into an ellipse, as we see in figure 3(b). In figure 3(c), at a higher driving frequency ($\omega_1 < \omega < 5.98$ rad s$^{-1} < \omega_2$), the Lissajous curve transforms to a negative slope, which means that two pendulums are in anti-phase synchronization. Increasing $\omega$ to 6.05 rad s$^{-1}$, the system returns to locked-phase synchronization, because an ellipse Lissajous curve appears again, as shown in figure 3(d). Finally, with sufficiently high driving frequency ($\omega = 7.94$ rad s$^{-1} > \omega_2$), the system returns to in-phase synchronization since a line of positive slope emerges once more in the Lissajous curve, as figure 3(e) shows. Above all, figures 3(a)-(e) display in a clear way the change of synchronized states (in-phase $\rightarrow$ locked-phase $\rightarrow$ anti-phase $\rightarrow$ locked-phase $\rightarrow$ in-phase synchronization) with increasing driving frequency.

The phase-difference frequency characteristic curve (PFC curve) of the system is depicted in figure 3(f). This curve shows the whole process of changing synchronized states. Through it, we can see that there are five state regions, including in-phase $(A, E)$, locked-phase $(B, D)$, and anti-phase $(C)$ synchronized states.

3.1.2. Effect of coupling strength. In order to analyse the effect of the coupling strength, we compare amplitude–frequency characteristic curves (AFC curves) of two pendulums ($l_1 = 0.275$ m, $l_2 = 0.245$ m) with different coupling strengths.

Without the coupling, the AFC curve of each pendulum only has one resonance peak, as shown in figure 4(a). A small convex appears on the AFC curve of each pendulum with the addition of magnets. The position of the small convex in the AFC curve of one pendulum is at almost the same position as the other pendulum’s resonance peak, as illustrated in figures 4(b) and (c). We believe that the pendulum of higher amplitude drives the other one to swing through the coupling magnets. When one pendulum’s amplitude reaches its resonance peak, the other one is still small. As we know, the larger the amplitude is, the more quickly the pendulum ball passes the equilibrium position. Therefore, the coupling effect is weakened. On the other hand, we must note the attraction between the magnets. As mentioned previously, when there is an angle between adjacent magnets, the attraction will enable adjacent magnets to return to the equilibrium position. But once this angle exceeds a certain value, the attraction will decrease. Consequently, the drive effect can be obviously reduced when the difference between the two pendulums’ amplitude is too large.

Apart from the phenomenon mentioned above, we can also observe the exchange of the two pendulums’ resonance peaks. The resonance peak of pendulum 2 (blue dashed line) is higher than that of pendulum 1 (red solid line) without coupling, as shown in figure 4(a). With two pairs of coupling, the resonance peak of pendulum 1 outruns that of pendulum 2, as illustrated in figure 4(b). This tendency becomes more manifest with three pairs of coupling, as shown in figure 4(c).

3.2. Research on three coupled pendulums

Since the objects of our experiment are the coupled non-identical pendulums, we doubt whether different arrays of them will give rise to differences in their motions. To investigate, we compare the AFC curves of different arrays. To simplify our description of array, we denote the longest pendulum as pendulum 1 and the shortest one as pendulum 3, and
Figure 4. AFC curves of coupled pendulums. (a) No coupling. (b) Two pairs of magnets. (c) Three pairs of magnets.

Figure 5. AFC curves of three coupled pendulums. (a) No coupling. (b) Array ‘123’. (c) Array ‘213’. (d) Array ‘132’.

$l_1 = 0.275$, $l_2 = 0.245$, $l_3 = 0.235$ represent the pendulum lengths. Array ‘213’ then means that the lengths of pendulums are $l_2$, $l_1$, $l_3$, from left to right. We carry out the experiment with three pairs of coupling magnets.

The AFC curves of different arrays without coupling are the same, as shown in figure 5(a). With three pairs of magnets, an obvious distinction emerges between them. In array 123, the resonance peak of pendulum 1 (red solid line) is the highest, the peak of pendulum 3 (green dot line) is second and the peak of pendulum 2 (blue dash line) is the lowest, as shown in figure 5(b). In array 213, the resonance peaks are all about 40°. When it comes to array 132, the resonance peak of pendulum 2 turns out to be the highest and that of pendulum 1
the lowest. Consequently, differences between different arrays of pendulums emerge with increasing coupling strength. The synchronized states in the three coupled pendulums may also reflect the results given in section 3.1.2.

4. Conclusion

In this work, we conduct a very interesting experiment with some simple equipment, including pendulums, steel magnets, a CCD camera, and a stepping motor and its controller. We realize a coupled system with a number of steel magnets, and design a data acquisition system with a CCD camera connected to a computer. With this data acquisition system, we find many interesting phenomena. For example, with increasing the driving frequency, the coupled system can show different types of synchronization, such as in-phase, locked-phase and anti-phase synchronization, and with increasing coupling strength, a small convex on the AFC curves exists. In the future, we will build a theoretical model to explain all these phenomena. On the other hand, coupled pendulums with greater pendulum numbers will be considered and studied through experiment. We would also like to study the dynamics of coupled pendulums, when steel magnets provide a repulsive force [11].

Clearly, all these findings will attract the interest of not only researchers in nonlinear dynamics, but also undergraduate students studying general physics. From this simple experiment, the results will provide undergraduate students with a direct and vivid approach to the study of complex dynamical behaviours in coupled systems, and stimulate their creativity and interest.

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References