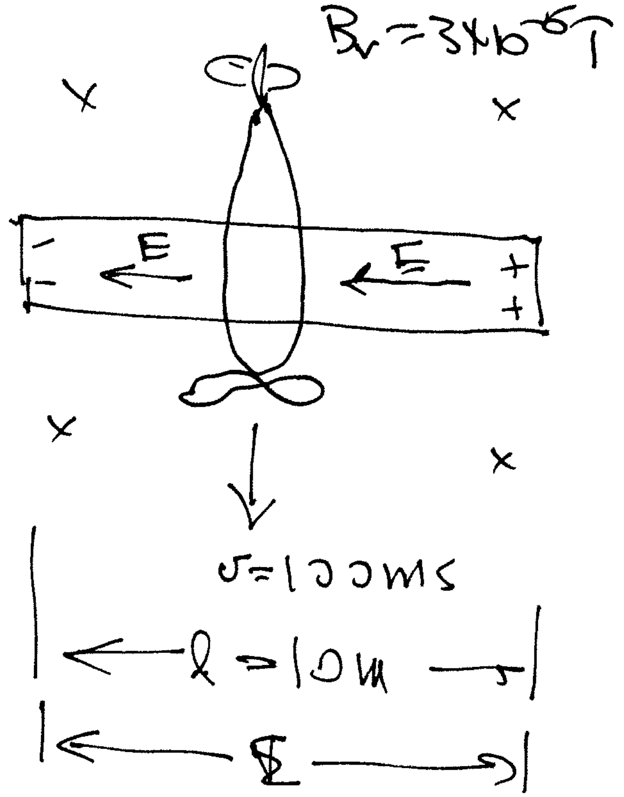
March 17 2007

Motional emf: airplane flies N at $v = 100\,m/s$

$B_{Earth} = 3\times10^{-5}\,T$ into the page (down)

$q\vec{v}\times\vec{B}$ is to right so ⊕ charges are on right, ⊖ on left



This is like Hall effect:

$$q\vec{v}\times\vec{B} + q\vec{E} = 0$$

$$vB = E = \mathcal{E}/\ell$$

$$E = \mathcal{E}/\ell \;(V/m)$$

$$\mathcal{E} = vB\ell = 100 \times 10 \times 3\times10^{-5}$$
(m/s, m, T)

$\mathcal{E} = .03\,V$ across the wings

Lenz's law: when the magnetic flux changes in a circuit, the induced $\mathcal{E}$, induced I, and induced B act to

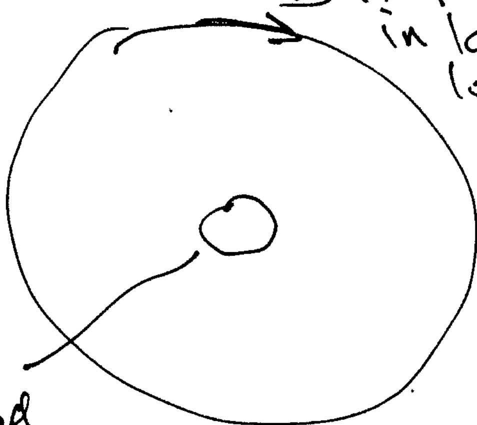
OPPOSE THE CHANGE IN FLUX

Induced ~~Induced~~ I, B, Φ_m act to preserve the ~~stat~~ status quo (Lenz's law)

Lenz's law examples

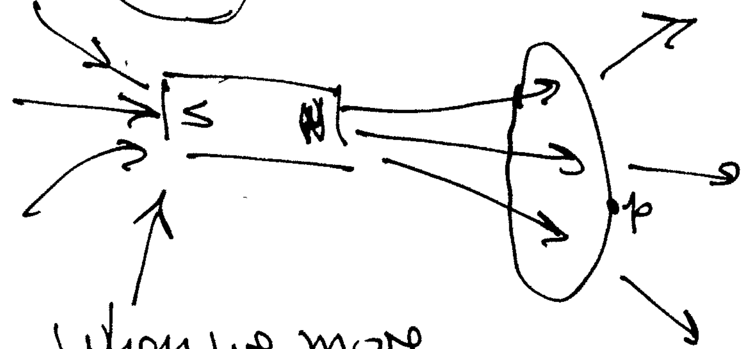
(A)

I increases in large loop



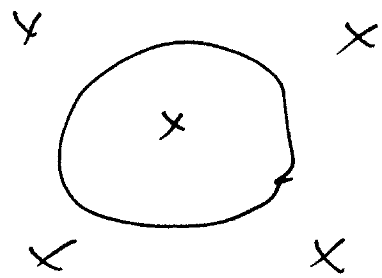
Is induced current cw or ccw

(B)



When we move the magnet AWAY FROM THE LOOP is the induced current at p up or down?

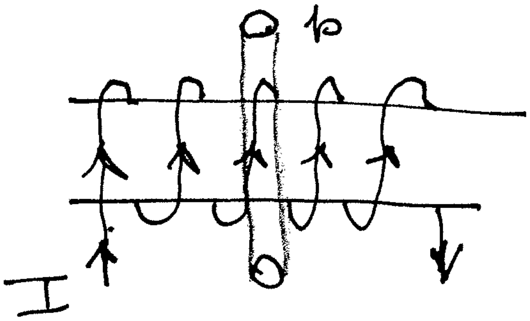
(C)



B is into the page and decreasing

Loop current is cw ccw

(D)



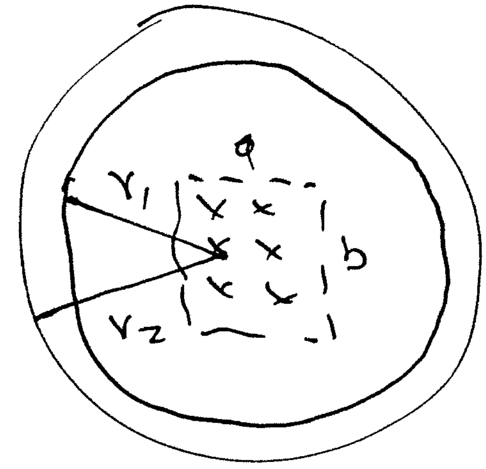
I in solenoid is INCREASING

In the loop around the solenoid, the current induced at p is IN OUT of the page

(A) : ccw (B) down (C) cw (D) out

p.1078 #24

B field is confined to a rectangular region ab.



Flux through loop of inner radius r_1 & outer radius r_2

$$\Phi_m = \int \vec{B}\cdot d\vec{A} = \int B\,dA\cos\theta$$

$$\Phi_m \neq B\pi r_1^2 \qquad \boxed{\Phi_m \neq B(\pi)\left(\frac{r_1+r_2}{2}\right)^2}$$

what is Φ_m? (see below)

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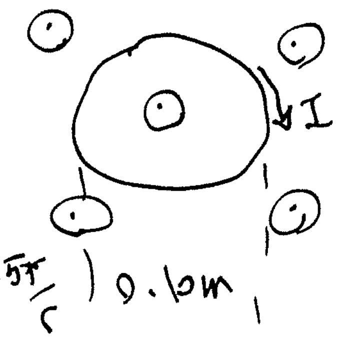
p.1077 #10 $R = 0.10\,\Omega$ for loop. loop diam $= 0.10$ m

a)



B is out

$\frac{dB}{dt} = +.5\,\frac{T}{s}$) 0.10m

B is increasing

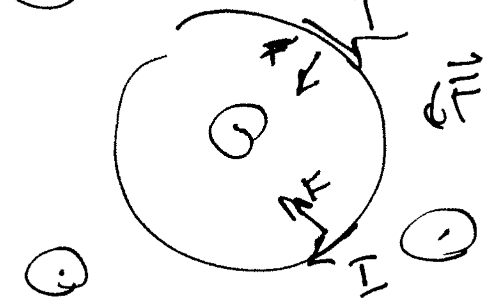
$$\Phi_m = B\pi r^2$$

$$\frac{d\Phi_m}{dt} = \frac{dB}{dt}\pi r^2 = |\mathcal{E}| = \left(\frac{1}{2}\frac{T}{s}\right)(\pi)(.05\text{m})^2$$

$$\mathcal{E} = .00393\text{ V} = 3.93\text{ mV}$$

$$I = \mathcal{E}/R = 39.3\text{ mA}$$

I is $\boxed{\text{cw}}$



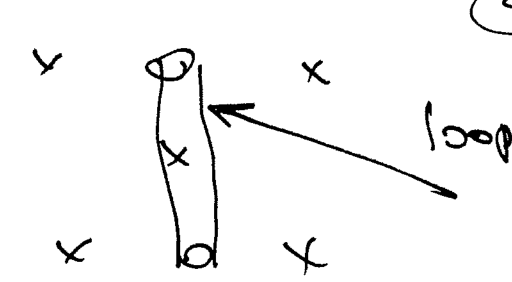
$d\vec{F} = I\,d\ell \times B$ toward the center

Force is toward the center, like for the crushed quarter

#24: $\Phi_m = B\cdot ab$

p.1077 c) There is no flux through the loop no $\mathcal{E}$, no I

(10)



B into the page

INDUCTANCE

$$\text{inductance} = \frac{\text{magnetic flux}}{\text{current}}$$

1 henry = $\frac{Tm^2}{A}$

$$L = \frac{\Phi_m}{I}$$

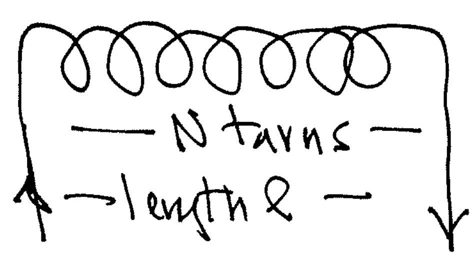
Φ_m ← flux in circuit

I ← current in that circuit

L = self inductance

Calculate L for a solenoid (p.1065)

solenoid



$n = \text{turns}/\text{length} = N/l$

$B_{sol} = \mu_0 n I$

$= \mu_0 \frac{N}{L} I$

$$\Phi_{m,\text{one turn}} = A_{sol} \cdot B_{sol}$$

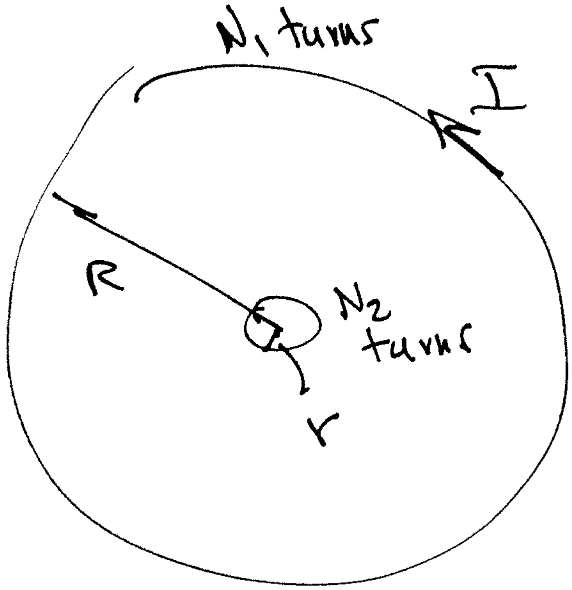
$$\Phi_{m,\text{one turn}} = A_{sol} \mu_0 \frac{N}{L} I$$

$$\Phi_{m,\text{entire sol}} = N(\Phi_{\text{one turn}}) = A_{sol} \mu_0 \frac{N^2}{L} I$$

$$L_{sol} = \frac{\Phi_{m\,sol}}{I} = A_{sol} \mu_0 \frac{N^2}{L} = L_{solenoid}$$

Calculate mutual inductance

$$M = \frac{\Phi_{circuit\ 1}}{I_{circuit\ 2}} = \frac{\Phi_{12}}{I_2} \Rightarrow \text{mutual inductance}$$



$$B_{\text{in small loop}} \doteq \frac{\mu_0 I}{2R} N_1$$

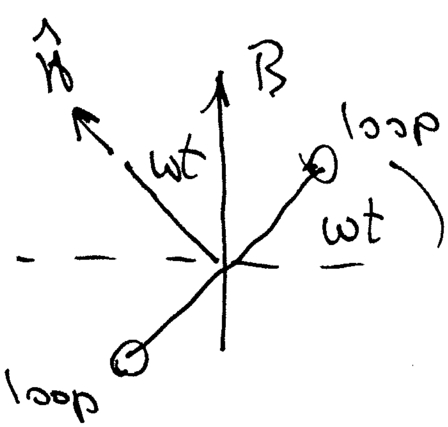
$$\Phi_{\text{small loop}} = \frac{\mu_0 I}{2R} \pi r^2 N_1 N_2$$

$$\boxed{M = \frac{\Phi_{12}}{I_1} = \frac{\mu_0 N_1 N_2 \pi r^2}{2R}}$$

More Faraday's law

$\mathcal{E}$ in <u>rotating</u> <u>loop</u> (p.1063)

θ between $\vec{B}$ & area vector of loop



$$\theta = \omega t$$

$$\Phi_m = AB\cos\omega t$$

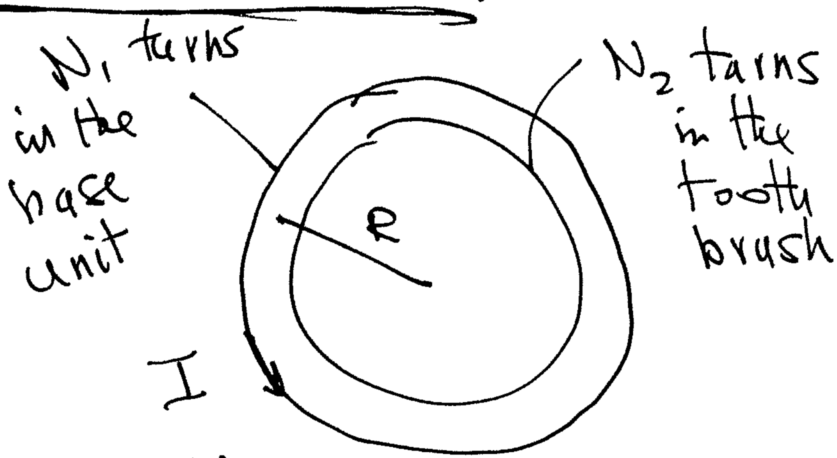
$$\mathcal{E} = \frac{d\Phi_m}{dt} = AB(-\omega \sin\omega t) \leftarrow \text{for 1 turn}$$

$$\mathcal{E}_{N\ \text{turns}} = N\omega AB \sin\omega t \quad \text{(rotating loop)}$$

(ω ← $2\pi f$)

Mon Faraday: **Electric toothbrush!**

current alternates in the base unit. Induced emf in the toothbrush charges the battery



$I = I_o \cos\omega t$ (base unit)

Take B in toothbrush (TB) to be $\frac{\mu_0 I}{2R}$
(It will be larger than this, but not too much.)

$$\Phi_{TB} = \left(N_1 \frac{\mu_0 I_o \cos\omega t}{2R}\right)\left(N_2 \pi R^2\right)$$

B A

(assumes both radii are the same)

$$\mathcal{E} = \frac{d\Phi_{TB}}{dt} = \frac{N_1 N_2 \mu_0 \pi R}{2} I_o(-\omega \sin\omega t)$$

let $R = .01\,m$ $I_o = 50\,mA$ $N_1 = N_2 = 500$

$\omega = 2\pi \cdot 60$

$$\mathcal{E}_{max}\ (\text{when } \sin\omega t = -1) = \frac{(500)^2(4\pi\times10^{-7})\pi(.01)(.05)(2\pi\cdot 60)}{2}$$

This will **not** charge the 1.5v battery!

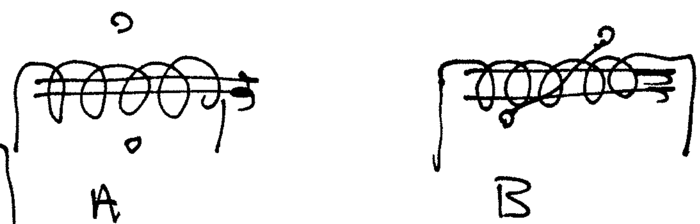
$\boxed{\mathcal{E}_{max} = .186\,v}$

(see next page)

Toothbrush: The frequency is actually 31,000 Hz
The designers jacked up the frequency by a factor of 500 so $\mathcal{E} \to (.186)500 = 93\text{V}!$

The induced emf is nothing like 93 volts. It's probably about 2 V.
I_0 may only be 2 mA and not 25 mA, and N_1 and N_2 are probably more like 250 than 500.

Problem 5, p. 1076



The flux through the loop is the same in A and B

The flux is the number of B field lines going through the loop. This is exactly the same, even if you tilt the loop

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Inductors in circuits

$$\mathcal{E} = -\frac{d\Phi_m}{dt}$$

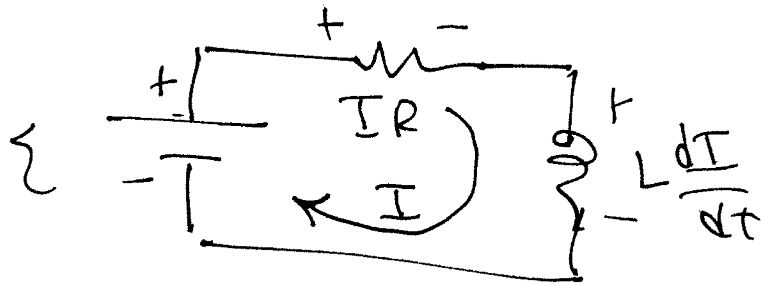
$$\frac{\Phi_m}{I} = L \text{ (inductance)}, \text{ or } \Phi_m = LI$$

$$\mathcal{E} = -\frac{d}{dt}(LI) = -L\frac{dI}{dt} = \text{emf across an inductor}$$

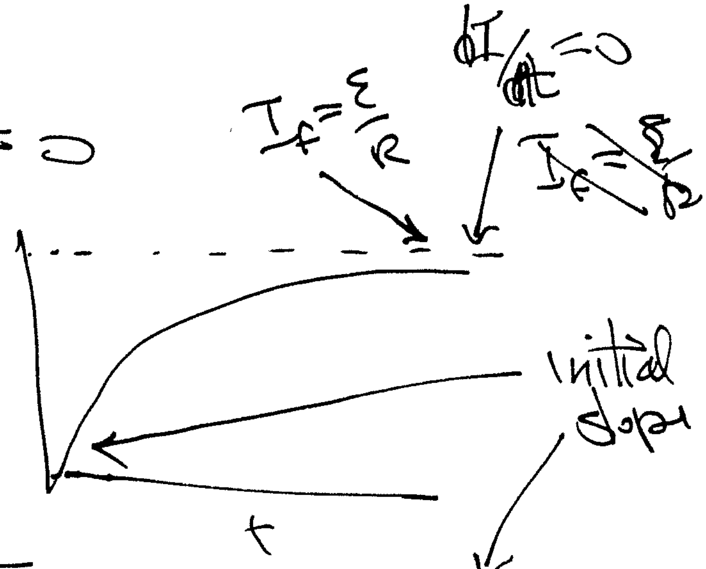
$$\mathcal{E} = -L\frac{dI}{dt}$$: emf across an inductor

This emf will oppose changes in current

Battery, resistor, inductor p. 1073



$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$



If we put the battery in at t=0
I will be zero at first

$$\mathcal{E} - (0)R - L\frac{dI}{dt}\Big|_0 = 0$$

$$\frac{dI}{dt}\Big|_0 = \mathcal{E}/L$$

If we have a current I and remove the battery at t=0:

$$0 - IR - L\frac{dI}{dt} = 0 \quad \text{(Eq 33.48)}$$

Rearrange

$$\frac{dI}{I} = -\frac{dt}{(L/R)}$$

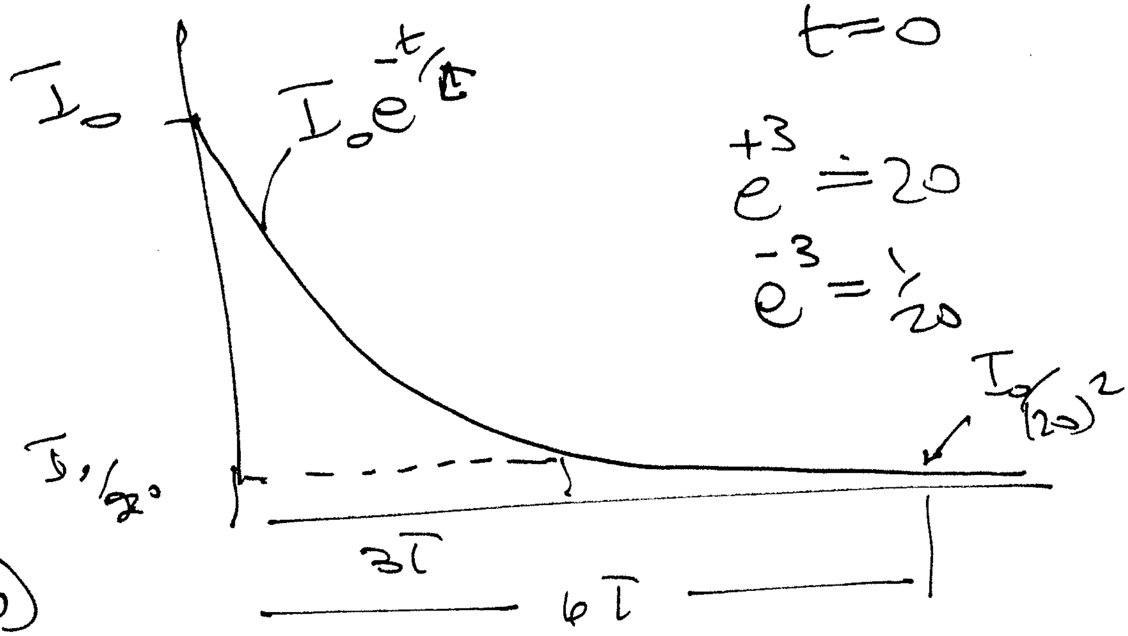
$L/R = \tau$ = time constant

$\frac{\text{henries}}{\text{ohms}}$ = seconds

$$\frac{dI}{I} = -\frac{dt}{\tau} \quad (\tau = L/R)$$

integrating we get $I = I_0 e^{-t/\tau}$

I_0 ← current at $t=0$



$e^{+3} \doteq 20$

$e^{-3} = 1/20$

After a few time constants (maybe 5-10) I is practically zero.

L/R dictates how fast current can increase or decrease in a circuit

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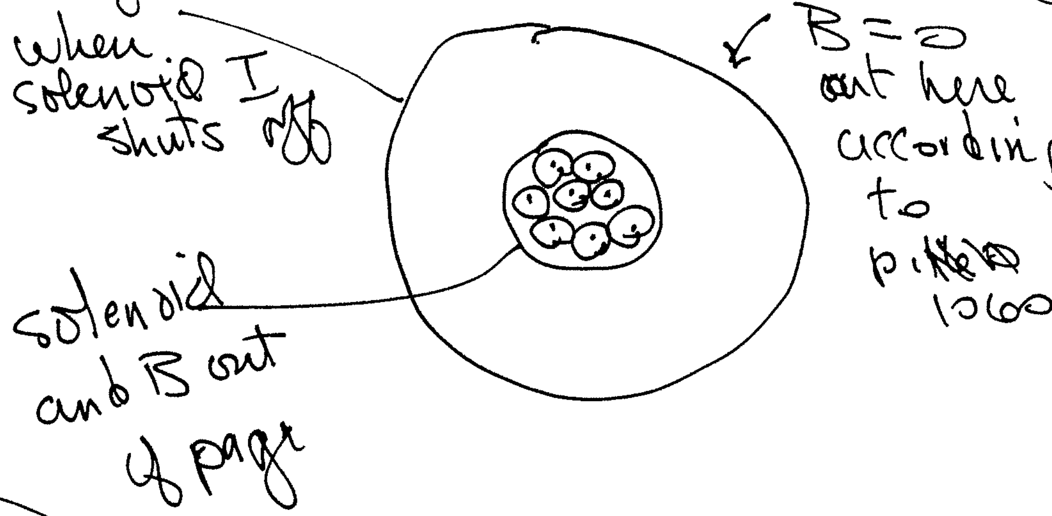
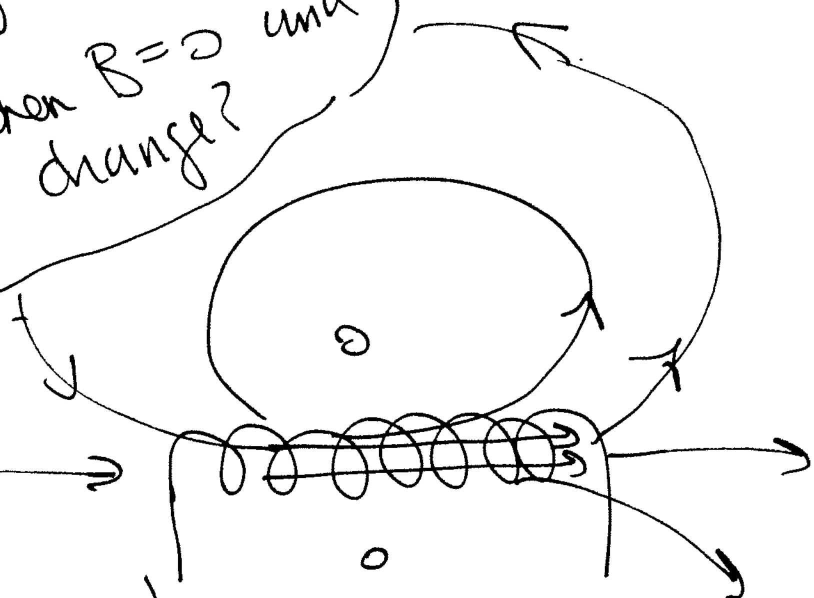
p. 1060 Get $\mathcal{E}$ in this loop



How do charges know to move when B = 0 and doesn't change?



All the lines in here have to go out and around and come in the back.

<u>B lines must loop around.</u>

When solenoid B collapses, all these B lines have to collapse too. As they collapse past the loop, we get $q\vec{v}\times\vec{B}$ and <u>this</u> is what creates $\mathcal{E}$.