Building Structures Mathematically in Minetest

Sonia Math Day – February 22, 2020
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Minetest – A Minecraft-Like Game

Minetest (minetest.net) is an open-source voxel game similar to Minecraft, and it is free to download! To start the game on the provided laptops,

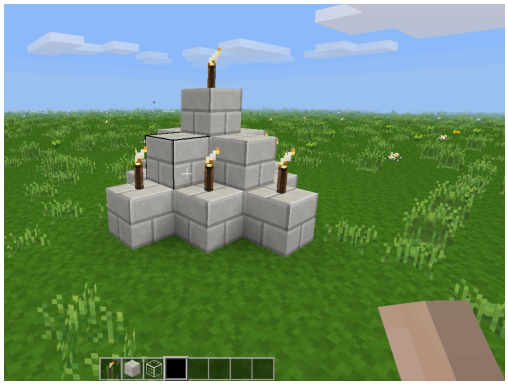
1. Navigate to

   `Desktop` → `Sonia Math Day minetest-5.1.1` → `bin` → double-click `minetest.exe`.

2. Select world `Sonia Math Day 2020 [minetest]`, then click `Play Game`.

Experiment for a few minutes! Here are some important controls:

Key / Mouse	Action
W	Move forward
S	Move backward
A	Move left
D	Move right
Spacebar	Jump / climb / fly up
Left-shift	Sneak / fly down
Left-click	Dig / punch
Right-click	Place block
Scroll wheel	Select item
I	Inventory screen
K	Enable / disable fly mode
J	Enable / disable fast mode
H	Enable / disable noclip mode (fly through walls)
ESC	Pause game
/time 6000	Set to daytime (6 AM)
/time 0	Set to nighttime (12 AM)



Specifications

Using mathematical equations, we will build a large "house" that has two main parts:

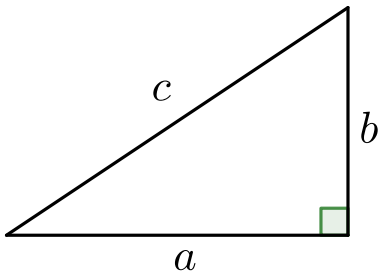
1. A cylindrical wall of radius 15 blocks and height 20 blocks.

2. A hemispherical dome that sits directly on top of the cylindrical wall.

Make a sketch:

The Pythagorean Theorem

For a right triangle, the *Pythagorean Theorem* provides a relationship between the lengths of the legs (shorter sides) and the length of the hypotenuse (longest side).

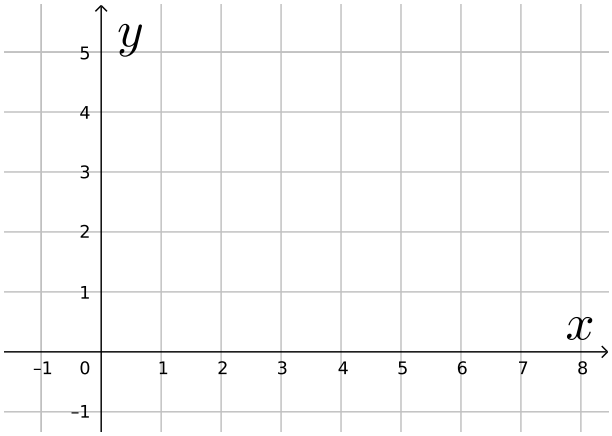


Equation:

While named for the Greek mathematician Pythagoras (circa 570-500 BCE), this theorem was known to the Babylonians as long ago as 1900 BCE! As we will see, the Pythagorean Theorem has many interesting consequences.

The Distance Formula

Question: How far apart are two locations?



(a) Plot the points $P(2, 4)$ and $Q(6, 1)$.

(b) Sketch a right triangle where segment $\overline{PQ}$ is the hypotenuse.

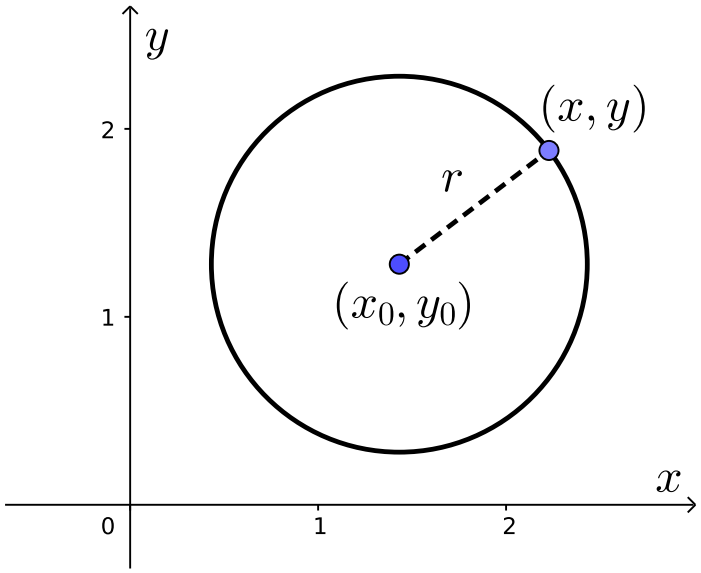
(c) Find the distance between P and Q.

In general, what is the distance between two points (x_0, y_0) and (x_1, y_1)?

Distance =

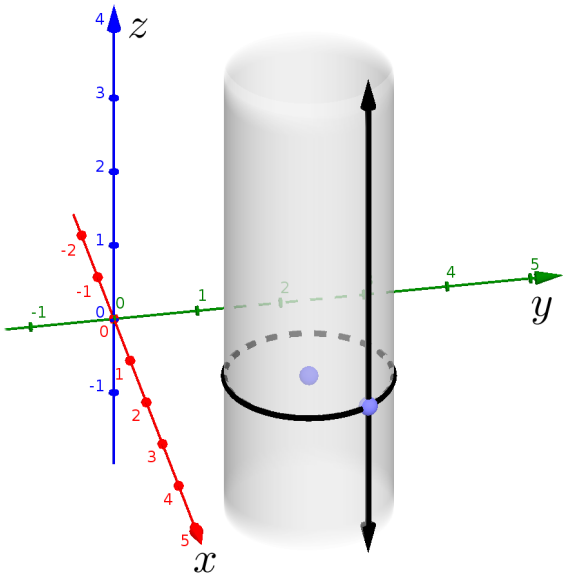
The Equation of a Circle

A *circle* is the set of all points in a plane that are *equidistant* (same distance) from a given point:



Equation:

The Equation of a Cylinder in 3D



The z-axis extends vertically out of the xy-plane to form three-dimensional space.

Then the set of points (x, y, z) such that

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

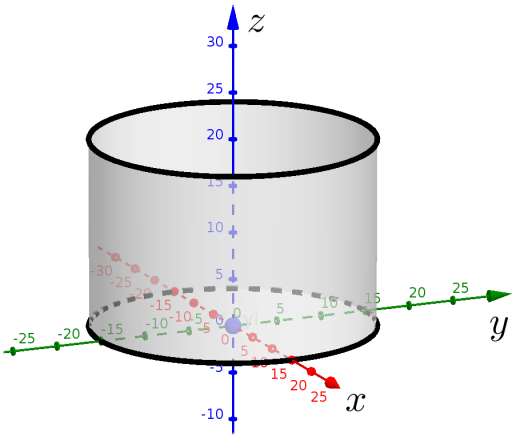
is an (infinite) cylinder!

For each point (x, y) on the circle in the xy-plane, **the z-coordinate is unconstrained**, meaning all points above and below the circle are part of the cylindrical surface!

Building the Cylindrical Wall

Let's set up the cylindrical wall of radius 15 blocks and height 20 blocks, centered about the z-axis!

Equation (in the form "stuff $= 0$"):



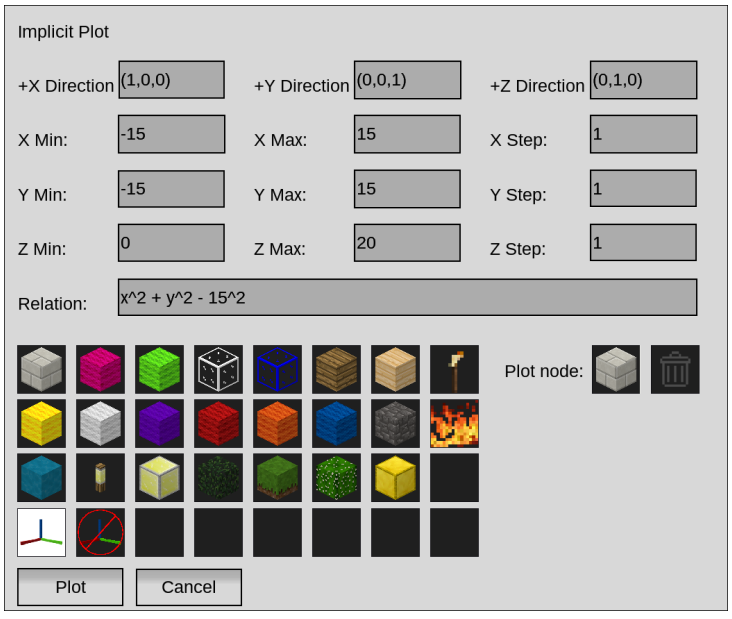
$\_\_\_\_\_ \leq x \leq \_\_\_\_\_$

$\_\_\_\_\_ \leq y \leq \_\_\_\_\_$

$\_\_\_\_\_ \leq z \leq \_\_\_\_\_$

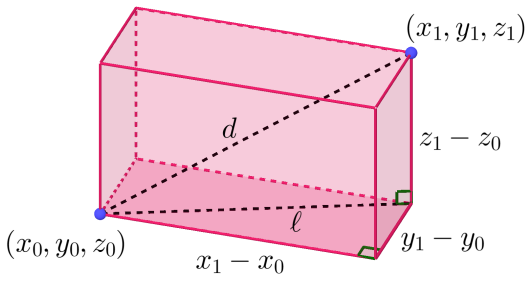
Now build it!

1. Punch (left-click) the origin node, which is white with the coordinate axes on them.
2. Click the `Implicit Plot` button.
3. Fill in `X Min`, `X Max`, `Y Min`, `Y Max`, etc. fields based on the ranges of the variables we found above. Just keep `1` for `X Step`, `Y Step`, and `Z Step`.
4. In the `Relation` field, type the left-hand side of the equation for the cylinder we found above. For exponents, use the caret: `^` (Shift-6). For example, `x^2` means x^2.
5. Drag the "Silver Sandstone Brick" into the `Plot node` box. (This is the type of block that will be used to draw the cylinder.)
6. Click the `Plot` button. (And marvel at your creation!)



Distance in 3D

We can apply the Pythagorean Theorem to find the distance between points in 3D space!



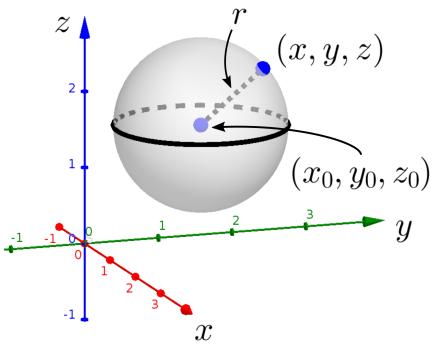
$d =$ distance between (x_0, y_0, z_0), (x_1, y_1, z_1)

$d^2 =$

The Equation of a Sphere

A *sphere* is the set of all points in space that are *equidistant* (same distance) from a given point:

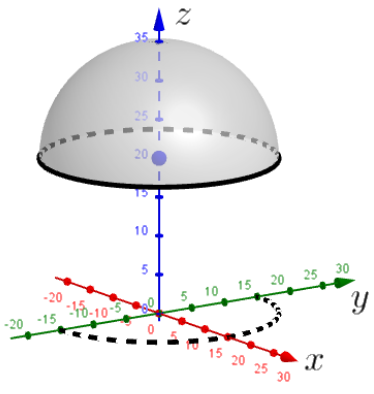
Equation:



Building the Hemispherical Dome

Radius: Center:

Equation (in the form "stuff = 0"):



_____ $\leq x \leq$ _____

_____ $\leq y \leq$ _____

_____ $\leq z \leq$ _____

In Minetest, use the `Implicit Plot` button, just like for the cylinder. Adjust the ranges, relation formula, and block type!

Further Additions

Floor for your cylindrical house

$$x^2 + y^2 \leq 15^2, \quad z = -1$$

```
X Min: -15, X Max: 15, Y Min: -15, Y Max: 15, Z Min: -1, Z Max: -1
Relation: x^2 + y^2 <= 15^2
```

Recommended block: a colored wool (carpet!)

Moat

$$27^2 \leq x^2 + y^2 \leq 40^2, \quad -10 \leq z \leq -1$$

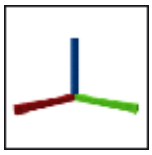
```
X Min: -40, X Max: 40, Y Min: -40, Y Max: 40, Z Min: -10, Z Max: -1
Relation: x^2 + y^2 >= 27^2 and x^2 + y^2 <= 40^2
```

Recommended block: River Water Source

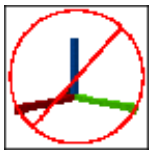
(Don't forget to build a bridge!)

Other `Implicit Plot` Examples

For these examples, you should set a new `Origin Node` relatively far away from the cylindrical house to avoid damaging it. In the inventory, an Origin Node looks like this:



Origin Node
This defines the origin of a new coordinate system.



Origin Node Destroyer
Equip this, then punch at an origin node to destroy it.

Pyramid

$$|x| + |y| + |z| - 40 = 0, \quad 0 \leq z \leq 40$$

```
X Min: -40, X Max: 40, Y Min: -40, Y Max: 40, Z Min: 0, Z Max: 40
Relation: abs(x) + abs(y) + abs(z) - 40
```

Recommended block: Desert Sandstone Brick

Tanglecube

$$\frac{x^4}{10000} - \frac{x^2}{20} + \frac{y^4}{10000} - \frac{y^2}{20} + \left(\frac{z}{10} - 2.5\right)^4 - 5\left(\frac{z}{10} - 2.5\right)^2 + 12 = 0$$

```
X Min: -25, X Max: 25, Y Min: -25, Y Max: 25, Z Min: 0, Z Max: 50
Relation: x^4/10000 - x^2/20 + y^4/10000 - y^2/20 + (z/10-2.5)^4 - 5*(z/10-2.5)^2 + 12
```

Recommended block: a colored glass