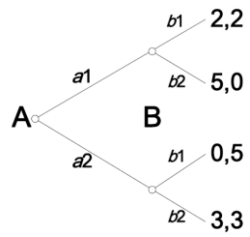
3.4: Game Theory as a Tool for Analyzing Strategic Interaction

To prepare for this lecture, read Hirschey, chapter 14, 549 -- 561.

Game theory is the study of how interdependent decision makers make choices. What game theory offers the manager is a "lens" through which he/she may formalize and analyze strategic decision making. To describe a strategic situation in game theoretic terms, rules of play must be explicit, and various "solution concepts" are used to "solve" a game (to find its ***equilibrium outcome***).

Formalizing a Game
There are two basic ways of laying out the basic features of a game: matrix (normal) form, and decision tree (extensive) form. Here is a simple game in both matrix (normal) and decision tree (extensive) form:

		B	
		*b*1	*b*2
A	*a*1	2,2	5,0
	*a*2	0,5	3,3



In this game there are two players (A and B), each of which have two strategies (A: *a*1 and *a*2; B: *b*1 and *b*2). The payoffs are functions of combined strategy choices. For example, is A chooses strategy *a*1 and B chooses strategy *b*1, then A earns 2 and B earns 2. If, on another hand, is A chooses strategy *a*1 and B chooses strategy *b*2, then A earns 5 and B earns 0.

Strategies and Solution Concepts
<u>Dominant Strategy</u>: A strategy that is superior to other strategies regardless of what strategy is chosen by an opposing player. In the game above, *a*1 is a dominant strategy for player A, while *b*1 is a dominant strategy for play B.
<u>Secure Strategy</u>: Also known as a maximin strategy, this is a strategy that guarantees the best outcome under the worst conditions. In the game above, the worst outcome for player A if he/she chooses strategy a1 is a payoff of 2. The worst outcome for player A is he/she chooses strategy *a*2 is a payoff of 0. The better of these outcomes is a payoff of 2. Therefore, strategy *a*1 is a secure strategy for player A. By similar reasoning, *b*1 is a secure strategy for player B.

In some cases, and the game above is one, an equilibrium outcome can be defined in terms of dominant strategies or secure strategies. In a game such as the one above, in which both players have a dominant strategy, the equilibrium outcome is clear, and we write the solution to this game as {*a*1, *b*1}. Not all games will be so easy to solve. Confirm that neither player has a dominant strategy in this game:

		B	
		b1	*b2*
A	*a1*	64,64	36,72
	a2	72,36	0,0

For this reason, it is common to employ more complex solution concepts. One of the most common is that of ***Nash Equilibrium***, which is a set of decision strategies in which no player can improve upon their payoff outcome through a unilateral change in strategy. Confirm that in the game above there are two Nash Equilibria.

The **Prisoner's Dilemma** is a famous game in which rational self interest leads to suboptimal outcomes for players. Here is a simple example in which pursuit of dominant strategies leads to an Nash Equilibrium that nevertheless is a suboptimal equilibrium outcome:

		B	
		$b1$	$b2$
A	$a1$	2,2	5,0
	$a2$	0,5	4,4

Economists have long studied strategic situations that resemble the prisoner's dilemma. Aggressive price competition may be one case in point, but excessive advertising expenditures by firms and the granting of production subsidies by governments may be others.

Sequential vs. Simultaneous Play

In some games, players move sequentially, responding to another player's prior moves. An incumbent firm that must decide how to respond to the entry into their market by a new competitor is one example. Such games are best analyzed by formalizing the game in extensive form and using a solution concept called ***backward induction***. The general idea, described by Hirschey as "look ahead and extrapolate back", is to work backwards on the decision tree of a game as a way of anticipating rational countermoves to initial decisions. We may also think of this process as "pruning" non-equilibrium outcomes to arrive at a ***subgame perfect Nash equilibrium***.

Some sequential games will have a first-mover advantage, a benefit enjoyed by the player who is able to make the initial move in a multistage game. Stackelberg competition, first discussed in lecture 3.2 is such a game.

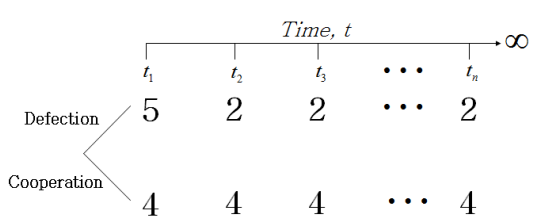
In other games, players move simultaneously, making their choices in an environment of incomplete information about the other player's strategy choice. A good example of this would be two firms that compete on price in different geographic markets or in the same market over many different time periods. Such games are best analyzed by formalizing the game in normal form and using Nash Equilibrium reasoning.

Repeated Play

Under certain conditions, ongoing interaction may change the nature of an equilibrium outcome because each player's payoff is now a stream of payoffs over time. This introduces the complicating factor of present value analysis. Taking a prisoner's dilemma as a starting point, under certain conditions players may be able to sustain the optimal outcome over many periods. As an example, in the prisoner's dilemma game above, suppose player A adopts the following strategy:

> *Play a2. If Player B plays b2, continue to play a2 (cooperation). If Player B plays b1 (defection), then play a1 until Player B changes back to cooperation.*

Notice that such a strategy (commonly referred to as "tit for tat" or a ***trigger strategy***) on the part of player A will mean that player B's stream of possible payoffs will look like this:



In evaluating this choice, player B must solve the following inequality:

$$\frac{5}{(1+\delta)^1}+\frac{2}{(1+\delta)^2}+\frac{2}{(1+\delta)^3}+\cdots+\frac{2}{(1+\delta)^n} \quad \gtreqless \quad \frac{4}{(1+\delta)^1}+\frac{4}{(1+\delta)^2}+\frac{4}{(1+\delta)^3}+\cdots+\frac{4}{(1+\delta)^n}$$

There will be some range of discount rates, δ, for which cooperation through time will be optimal. Of course, if either player perceives that the game is going to end in some foreseeable time period, it becomes more difficult to sustain cooperation as an equilibrium outcome (why?). This is what is known in game theory as the ***end-of-game problem***.

In infinitely repeated games, reputation takes on real economic value. This is most clear in so-called product quality games (see Hirschey, p. 558).

Relevant Textbook Problems: 14.2, 14.3, 14.4, 14.5, 14.6