

2.6: Time Series Econometrics

To prepare for this lecture, read Hirschey, chapter 6, 203 – 209, 217 – 225, and the case “Forecasting Global Performance for a Mickey Mouse Organization”

All economic time series may be thought of as exhibiting any or all of the following patterns: trend, cyclicity, seasonality, and randomness. If one knew the “equations of motion” that generate a time series, then forecasting its future path would be effortless. When confronted with a seemingly random time series, a useful way of thinking about trend analysis and projection is in terms of trying to uncover or “tease out” underlying trends, cycles, and seasonality. The econometric process thus seeks to establish mathematical formulations that describe the combined effects of these patterns. These mathematical formulations then become the basis for projection. A good forecast leaves nothing but the randomness (a.k.a. “white noise”) unexplained.

Trends

We can employ at least four models in attempting to fit a trend to data:

1. Deterministic linear trend, in which a dependent variable is simply taken to be a function of time. Thus, we regress the time series in question on a simple time variable, $t = 1, 2, 3, \dots$

$$y_t = a_0 + a_1 \text{time}_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma)$$

2. Deterministic polynomial trend, in which the relationship between a dependent variable and time is assumed to be more complex. The simplest form is quadratic, although higher order polynomials may be employed.

$$y_t = a_0 + a_1 \text{time}_t + a_2 \text{time}_t^2 + \dots + a_n \text{time}_t^n + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma)$$

3. Growth Curves (also known as S-curves). These are sometimes called population growth curves or market penetration curves. Such curves initially increase at an increasing rate, pass an inflection point, then increase at a decreasing rate toward some maximum value. One simple example is the Gompertz curve:

$$y_t = \alpha e^{-\beta e^{-\gamma t}} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma)$$

4. Stochastic trend, in which a dependent variable is simply taken to be a function of its own past values:

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma)$$

Cycles

The econometric approach to incorporating cyclicity into a forecasting model involves finding a business cycle indicator, such as **capacity utilization rates** or **industrial production rates**, with which a dependent variable has been correlated in the past. Of course, incorporating such a measure into a forecasting model means that the forecaster must develop or find a forecast of the correlated variable. Consensus forecasts (discussed in lecture 2.5) often provide such forecasts.

Seasonality

Most economic time series exhibit some seasonality, usually quarterly or monthly. One common econometric method for incorporating seasonality into a forecasting model involves the use of **dichotomous or “dummy” variables** to generate **seasonal factors**. These seasonal factors measure the regular difference in the dependent variable from some arbitrarily selected base period (a specific month or quarter). The figure below illustrates the use of dummy variables to capture seasonality in a stochastic trend model. Note that if the seasonal frequency is n ($n = 12$ for monthly data, $n = 4$ for quarterly data), then the number of dummy variables incorporated into the econometric model is $n - 1$ (11 for monthly data, 3 for quarterly data).

Model Specification With Monthly Dummy Variables:

$$y_t = a_0 + a_1 y_{t-1} + a_i \sum_{i=2}^{12} D_i + \varepsilon_t$$

where

$$D_2 = \begin{cases} 1 & \text{if February} \\ 0 & \text{otherwise} \end{cases}$$

$$D_3 = \begin{cases} 1 & \text{if March} \\ 0 & \text{otherwise} \end{cases}$$

$$\vdots$$

$$D_{12} = \begin{cases} 1 & \text{if December} \\ 0 & \text{otherwise} \end{cases}$$

Data Matrix for Monthly Dummy Variables:

	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12
Jan-00	0	0	0	0	0	0	0	0	0	0	0
Feb-00	1	0	0	0	0	0	0	0	0	0	0
Mar-00	0	1	0	0	0	0	0	0	0	0	0
Apr-00	0	0	1	0	0	0	0	0	0	0	0
May-00	0	0	0	1	0	0	0	0	0	0	0
Jun-00	0	0	0	0	1	0	0	0	0	0	0
Jul-00	0	0	0	0	0	1	0	0	0	0	0
Aug-00	0	0	0	0	0	0	1	0	0	0	0
Sep-00	0	0	0	0	0	0	0	1	0	0	0
Oct-00	0	0	0	0	0	0	0	0	1	0	0
Nov-00	0	0	0	0	0	0	0	0	0	1	0
Dec-00	0	0	0	0	0	0	0	0	0	0	1
Jan-01	0	0	0	0	0	0	0	0	0	0	0
.
.
.
Dec-XX	0	0	0	0	0	0	0	0	0	0	1

[?] Think about how you would interpret (give meaning to) the parameter estimates for such dichotomous or “dummy” variables.

Serial Correlation in Economic Time Series

In classical ordinary least squares regression, the model assumes that the disturbance term relating to any one observation is not influenced by the disturbance term relating to any other observation (assumption #2 first mentioned in lecture 2.4). In economic time series this assumption is often violated. For example, if we are dealing with monthly data on the demand for building materials, a hurricane in one period (an exogenous or random “shock” in the data) may have lasting effects on subsequent periods as rebuilding occurs (the shock has “persistence”). This would mean that a stochastic trend would have the following property:

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t = \rho \varepsilon_{t-1} + \mu_t, \text{ where } \rho \neq 0 \text{ and } \mu_t \sim N(0, \sigma)$$

The practical difficulty introduced by serial correlation is inflation of standard errors of parameter estimates. It will also mean that a stochastic trend will tend to “wander”. Detection of serial correlation is difficult when using MSExcel to perform regression analysis. Standard econometric software such as EViews, Stata, or Minitab, however, all have relatively simple detection and correction procedures.

Exponential Smoothing

Hirschey’s discussion (pages 217 – 220) is self-explanatory.

Relevant Textbook Problems: 6.7, 6.8, 6.9, 6.10