

1.6: Cost Analysis and Topics in Production Theory

To prepare for this lecture, read Hirschey, chapter 8.

This lecture addresses five topics:

1. Mapping costs to variations in output – the “shape” of short-run cost functions
2. Empirical estimation of cost functions
3. Economies of scale
4. Extension to multi-plant operation.
5. Operating leverage

The “Shape” of Cost Functions

We now consider costs as a function of variations in output, q – in other words we map a function of cost over a range of outputs. This is how we normally think of the cost function. At the most general level of abstraction, we normally assume that this function is monotonically increasing in q – but the nature of that increase is open to investigation. It may be linear or it may take on some more complex shape. It is convenient to model these shapes as polynomials of n degree. Some examples:

Linear: $C(q) = a + bq$ (Marginal costs are constant – implications?)

Quadratic: $C(q) = a + bq + cq^2$ (Marginal costs are increasing linearly – implications?)

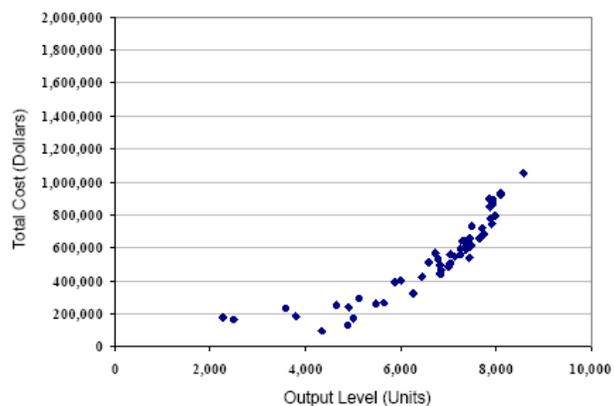
Cubic: $C(q) = a + bq - cq^2 + dq^3$ (Marginal costs are quadratic)

If there are *diminishing marginal returns* in the underlying production process and if inputs are purchased in competitive markets (two very strong assumptions), then the cost function will be of the second or third form.

Empirical Estimation of Cost Functions

Given enough historical data on costs and output, it is possible to empirically estimate a cost function by fitting a line to the data¹:

WEEK ENDING	WORKING DAYS	TOTAL PRODUCTION	VARIABLE COST
2-Jan-04	1	2,241	\$175,904
9-Jan-04	5	7,694	\$716,930
16-Jan-04	5	7,748	\$684,745
23-Jan-04	5	7,873	\$851,112
30-Jan-04	5	7,263	\$562,742
6-Feb-04	5	7,484	\$615,838
13-Feb-04	5	6,980	\$488,814
:	:	:	:
:	:	:	:
19-Nov-04	5	7,862	\$893,991
26-Nov-04	3	7,942	\$867,141
3-Dec-04	5	7,948	\$886,088
10-Dec-04	5	8,099	\$921,593
17-Dec-04	5	7,041	\$506,186
24-Dec-04	4	4,914	\$236,556
31-Dec-04	4	4,998	\$174,230



A precisely estimated cost function can allow us to make inferences about marginal costs. As we will see when we discuss pricing, most companies tend to use some form of a “cost-plus” pricing formula in which

¹ Statistical tools for doing this are the subject of lecture 2.3.

they mark up *average* production costs. But profit maximization presupposes the equality of marginal revenue and *marginal* costs, not average costs. If the total cost function is nonlinear, marginal costs may be rising with output, and thus average costs can be a very poor proxy for marginal costs. In other words, depending on the nature of the cost function, cost-plus pricing based on *average* costs may not be a profit-maximizing strategy.

Economies of Scale

Constant returns to scale exist when output changes are proportional to input changes (changing all inputs). **Increasing returns to scale** are prevalent if an increase in output resulting from an increase in all inputs is proportionally larger than the underlying increase in inputs. If input costs are constant over the increase in input usage, increasing returns to scale will result in **economies of scale** – a situation in which long-run average total costs decline as output levels increase.²

The production tables below illustrate both cases for a **cobb-douglas production function**. Notice that in the upper panel, output increases disproportionately along the diagonal, while in the lower panel output increases proportionally to the increase in inputs. Notice further that even with increasing returns to scale in the long run, there are diminishing marginal returns in the short run. This is illustrated by examining the changes in output horizontally or vertically, either of which implies holding one of the factors fixed. In both panels, the boldface output levels illustrate the effect of holding input x_1 fixed at a level of 3.

$q = f(x_1 = 3, x_2) = x_1^\alpha x_2^\beta$		Units of input x_2					
		1	2	3	4	5	
Increasing Returns to Scale $\alpha = \beta = 0.6$	Units of input x_1	1	1.00	1.52	1.93	2.30	2.63
	2	1.52	2.30	2.93	3.48	3.98	
	3	1.93	2.93	3.74	4.44	5.08	
		$dq/dx_2 = mp_2$	1.00	0.81	0.70	0.64	

		Units of input x_2					
		1	2	3	4	5	
Constant Returns to Scale $\alpha = \beta = 0.5$	Units of input x_1	1	1.00	1.41	1.73	2.00	2.24
	2	1.41	2.00	2.45	2.83	3.16	
	3	1.73	2.45	3.00	3.46	3.87	
		$dq/dx_2 = mp_2$	0.72	0.55	0.46	0.41	

Economics of Multi-Plant Operation

Let's say you can operate a plant with this cost function:

$$\text{Total Costs} = C(q) = 250,000 + 40q + 0.01q^2$$

² Economies of scale should not be confused with **economies of scope**, in which the joint costs of producing two goods are less than the costs of producing the goods separately. Economies of scope usually arise when there is some production complementarity across two or more goods – when production processes can easily be applied to the production of more than one good. See Hirschey's discussion on pages 313 – 314.

Thus, Marginal Costs = $dC(q)/dq = 40 + 0.02q$.

Assume demand is described by $P = 940 - 0.02q$; therefore $MR = 940 - 0.04q$. Hence, optimal output is $q^* = 15,000$ and profit is $\pi = \$6.5$ million.

Let's look at this outcome in a bit more detail by first examining the behavior of average costs. We can tell from the marginal cost function (which is rising), but what about average costs? As it turns out, at the optimal output, $ATC = 207$ ($TC = 3.1$ million / $15,000 = 206.667$). But minimum ATC is 140:

$$ATC = C(q)/q = 250,000q^{-1} + 40 + 0.01q$$
$$dATC/dq = -250,000q^{-2} + 0.01 = 0, q^* = 5,000, ATC = 140.$$

This implies that at the profit-maximizing output level average costs are rising. What if we could operate at (or near) minimum average total costs? This changes the profit-maximizing output level:

$$MR = 940 - 0.04q = 140 (= MC), q^* = 20,000 \dots \text{divided across four plants.}$$

Profits under this scenario are 8 million.

General lessons

1. If profit-maximizing operation requires producing on the upward sloping portion of ATC, there may be opportunities for profit improvement by operating multiple, smaller plants.
2. This assumes identical plants. If plants are similar, but not identical, the general rule is $MR = MC_1 = MC_2 = \dots = MC_n$

Operating Leverage

Operating leverage is a measure of how sensitive profits are to changes in sales. If operating leverage is high, a small percentage increase in sales can produce a much larger percentage increase in profit. It is high near the breakeven point and decreases as the sales and profit increase. The more important are fixed costs in a firm's cost structure, the greater the operating leverage. A business that has a higher proportion of fixed costs and a lower proportion of variable costs is said to have used more operating leverage.

The degree of operating leverage may usefully be thought of as the elasticity of profits with respect to output and may be calculated for any level of output:

$$DOL = \frac{d\pi/\pi}{dq/q} = \frac{dq(P - AVC)/[q(P - AVC) - TFC]}{dq/q} = \frac{q(P - AVC)}{q(P - AVC) - TFC} = \frac{P - AVC}{P - ATC}$$

Like financial leverage, operating leverage magnifies results, making gains look better and losses look worse. Both operating and financial leverage increase risks because they make returns less predictable over time.

Relevant Textbook Problems: 7.4, 7.10, 8.6, 8.7, 8.8, 8.9, 8.10