

1.2: Value Maximization

To prepare for this lecture, read Hirschey, case study “Is Coca-Cola the ‘Perfect’ Business?”; chapter 1, pages 6 – 8, 14 – 15; chapter 2, appendices 2A and 2B; and appendix A (pages 787 – 802).

Value Maximization

Let’s examine the mathematics of a simple (static or one-period) maximization problem for the firm:

$$\max \pi(q), \text{ where } \pi(q) = R(q) - C(q) \text{ and } q = f(x_1, x_2, \dots, x_n) \text{ with input prices } v_1, v_2, \dots, v_n$$

For a price-taking firm¹, this is

$$\max \pi(q), = R(q) - C(q) = p \cdot f(x_1, x_2, \dots, x_n) - [x_1 v_1 + x_2 v_2 + \dots + x_n v_n]$$

Setting this up as a Lagrangian ...

$$L = p \cdot f(x_1, x_2, \dots, x_n) + \lambda [C_0 - v_1 x_1 - v_2 x_2 - \dots - v_n x_n]$$

The first order conditions for minimization are ...

$$p f_1 - v_1 = 0; p f_2 - v_2 = 0; \dots; p f_n - v_n = 0$$

where $f_i = MP_i$, and from which we can easily observe a condition necessary for optimal input usage:

$$\frac{MP_1}{v_1} = \frac{MP_2}{v_2} = \dots = \frac{MP_n}{v_n}$$

? How would you interpret this maximization condition?

The foregoing is a static view of the firm’s optimization problem. Firms don’t exist for only one period and this realization complicates our view of the firm’s profit function (but we still maintain the assumption that the profit function is the objective function of the firm and that management’s job is to maximize it!). Here is an objective function that incorporates the notion of time:

$$\max_q \pi(q) = \int_{t=0}^{\infty} \pi(q) dt = \int_{t=0}^{\infty} [R(q) - C(q)] dt$$

This way of thinking about the firm’s objective function raises at least three complicating issues:

1. Accounting profit vs. economic profit. This isn’t actually a new issue – this distinction is equally important for a static or dynamic conception of the profit function.²
2. Profit today vs. profit in the future. This distinction underlines the importance of the **time value of money**.
3. Profit from different activities. If risk is a cost (or an economic “bad”), then equal profits with unequal risks must be treated differently. In finance terms, we must “risk-adjust” profits for comparison purposes.

¹ Notice that we are dealing with a simplification here: the firm is assumed to produce only one product.

² Hirschey (page 9) uses the terms **business profit** and **accounting profit** interchangeably.

Lecture 1.4 addresses issue 1. We'll postpone a full discussion of issue 3 until lectures 4.1 through 4.4. For present purposes, let's look more deeply at issue 2.

Time Value of Money

The **present value** (*PV*) of an amount to be received at the end of n periods (a **future value**, *FV*) when the per-period discount rate is i , is:

$$PV = FV/(1 + i)^n, \text{ and conversely, } FV = PV(1 + i)^n$$

The present value (*PV*) of a *stream of future values* (or an *uneven series of receipts*)³, when the per-period discount rate is i , is:

$$PV = \frac{FV_0}{(1+i)^0} + \frac{FV_1}{(1+i)^1} + \dots + \frac{FV_n}{(1+i)^n} = \sum_{t=0}^n \frac{FV_t}{(1+i)^t}$$

If we think of the future values as the future profits of the firm, then this last formula reduces to Hirschey's "basic valuation model" (page 6):

$$V = \sum_{t=0}^n \frac{\pi_t}{(1+i)^t}$$

A slightly more complicated version of this approach, and one building on the dynamic objective function introduced in lecture 1.1, is:

$$\max_q \pi(q) = \int_{t=0}^{\infty} e^{it} \pi(q) dt = \int_{t=0}^{\infty} e^{it} [R(q) - C(q)] dt$$

where e^{it} is a **discount factor**. In this case, we are considering the case of **instantaneous compounding**. In general,

$$FV = PV(1 + i/n)^{nt}, \text{ where } \begin{cases} i = \text{interest rate} \\ n = \text{compounding periods} \\ t = \text{years} \end{cases}$$

Consider \$100 ($PV = \100) to be compounded annually ($n = 1$) at 10% ($i = 0.1$) for 5 years ($t = 5$):

$$FV = 100(1 + 0.1/1)^{1*5} = 163.051$$

Now consider \$100 ($PV = 100$) to be compounded monthly ($n = 12$) at 10% ($i = 0.1$) for 5 years ($t = 5$):

$$FV = 100(1 + 0.1/12)^{12*5} = 164.531$$

If the compounding were instantaneous ($n \rightarrow \infty$),

$$FV = \lim_{n \rightarrow \infty} PV(1 + i/n)^{nt} = \lim_{n \rightarrow \infty} PV[(1 + i/n)^{n/i}]^{it} = \lim_{n \rightarrow \infty} PV[(1 + 1/m)^m]^{it}, \text{ where } m = n/i$$

$$\lim_{m \rightarrow \infty} PV[(1 + 1/m)^m]^{it} = e^{it}$$

³ Note that an **annuity** is a series of fixed or even payments for a specified number of periods.

\therefore with instantaneous compounding, $FV = PV * e^{it}$

Two Relevant Questions

Two other questions we might ask at this point are:

1. Why do firms exist? This is a question most famously ask by Nobel laureate Ronald Coase in his 1937 paper “The Nature of the Firm”.⁴ His answer – because markets do not function costlessly – led to the development of transaction cost economics, a subject taken up by Hirschey in chapter 18, and which we will address in lecture 4.5.
2. Do firms have any responsibility other than value maximization for the owners? Nobel laureate Milton Friedman provided a definitive (and controversial) answer to this question when he wrote:

“Few trends could so thoroughly undermine the very foundations of our free society as the acceptance by corporate officials of a social responsibility other than to make as much money for their stockholders as possible.”⁵

Relevant Textbook Problem: 2B.1

⁴ Ronald Coase, “The Nature of the Firm”, *Economica* 4 (1937), 386 – 405.

⁵ Milton Friedman, “The Social Responsibility of Business is to Increase its Profits”, *New York Times Magazine*, September 13, 1970.