Numerical Solutions and Direction Fields for Systems

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A worksheet to illustrate how to solve systems of ODEs numerically, and to draw a direction field for a pair of autonomous ODEs in Maple.

Load in DEtools package, which contains the helpful ***DEplot*** command, and the plots package, which contains ***odeplot***.

```
> with(DEtools) :
  with(plots) :
```

Example: A pair of ODEs:

```
> de1 := x1'(t) = x1(t) - x2(t)^2;
  de2 := x2'(t) = x1(t)·x2(t) + x1(t)
```

Numerical Solution: We can solve the system numerically with initial data x1(0) = 4, x2(0) = 1, with the command

```
> nsol := dsolve({de1, de2, x1(0) = 4, x2(0) = 1}, {x1(t), x2(t)}, numeric)
```

The solution at a given time t, say t = 1, can be obtained with

```
> nsol(1)
```

The resulting solution components can be plotted for -2 < t < 2 using the ***odeplot*** command

```
> odeplot(nsol, [[t, x1(t)], [t, x2(t)]], -2 ..2, color = [red, blue])
```

Or we can plot (x1(t), x2(t)) as a parametric curve:

```
> odeplot(nsol, [x1(t), x2(t)], -2 ..2)
```

The numerical solution procedure of ***dsolve*** works on systems of any dimension, as does the plotting of solution components xj(t) versus t. Parametric plotting of solution curves works in 2 and 3 dimensions.

Direction Fields: If the ODEs are autonomous, we can sketch a direction field on a range a < x1 < b, c < x2 < d, by using the ***DEplot*** command. We must supply a range for the independent variable "t", even though that range is not explicitly used in drawing this direction field.

```
> DEplot([de1, de2], [x1(t), x2(t)], t = 0 ..2, x1 = -5 ..5, x2 = -5 ..5)
```

Below is the same plot but with solution curves that pass through the points (x1,x2) = (4,1) and (x1,x2) = (3,-3), at time t = 0. The curves are sketched for t = 0 to t = 2 here.

```
> DEplot([de1, de2], [x1(t), x2(t)], t = 0 ..2, x1 = -5 ..5, x2 = -5 ..5, [[x1(0) = 4, x2(0) = 1],
      [x1(0) = 3, x2(0) = -3]], linecolor = black)
```

```
>
```