

# LSD Pharmacokinetic Model

Kurt Bryan and SIMIODE

A notebook to start the analysis of the two-compartment pharmacokinetic LSD model in Section 6.5.1.

**The Data:** Sampling times, in hours

```
In[1]:= times = {1/12, 1/4, 1/2, 1, 2, 4, 8}
```

The concentration data (mg/liter) for each subject at the seven times above is shown below (with "0" for the missing data point at time  $t = 4$  hours for subject 3).

```
In[2]:= concdata = {{11.1, 7.4, 6.3, 6.9, 5, 3.1, 0.8},
                 {10.6, 7.6, 7, 4.8, 2.8, 2.5, 2}, {8.7, 6.7, 5.9, 4.3, 4.4, 0, 0.3},
                 {10.9, 8.2, 7.9, 6.6, 5.3, 3.8, 1.2}, {6.4, 6.3, 5.1, 4.3, 3.4, 1.9, 0.7}};
```

A plot of the subject concentration data. First amalgamate data in to 5 lists of (times, concentration) data for each subject

```
In[3]:= concdata2 = Table[Table[{times[[j]], concdata[[i, j]], {j, 1, 7}}, {i, 1, 5}];
```

```
In[4]:= ListPlot[concdata2, PlotMarkers → Automatic,
                PlotLegends → {"Subject 1", "Subject 2", "Subject 3", "Subject 4", "Subject 5"}]
```

We use  $c_P(t)$  for the plasma concentration and  $c_T(t)$  as the tissue concentration we have  $c_P(0) = 12.27$  for each subject and  $c_T(0) = 0$ .

**The ODE Model:** The model of Section 6.5.1 (equation (6.66)) is

```
In[5]:= deP1 = 0.163 * cP'[t] == 0.115 * ka * cT[t] - 0.163 * kb * cP[t] - 0.163 * ke * cP[t];
deT1 = 0.115 * cT'[t] == 0.163 * kb * cP[t] - 0.115 * ka * cT[t];
```

where  $k_a$ ,  $k_b$ , and  $k_e$  are the constants to be determined from the concentration data.

As noted after equation (6.66), we can fill in the expressions

```
In[7]:= VP = 0.163 * M;
VT = 0.115 * M;
```

into the ODEs "deP1" and "deT1" above and find that "M" cancels. We are left with

```
In[9]:= deP = Assuming[M > 0, Simplify[MultiplySides[deP1, 1/M]]]
deT = Assuming[M > 0, Simplify[MultiplySides[deT1, 1/M]]]
```

These are equations (6.67) in the text.

We can have Mathematica solve these ODEs symbolically with initial data  $c_P(0) = 12.27$  and  $c_T(0) = 0$ . The resulting solution is quite complicated, so the display is suppressed by putting a semicolon after DSolve (remove it if you want to see the solution.)

```
In[11]:= sol = DSolve[{deP, deT, cP[0] == 12.27, cT[0] == 0}, {cP, cT}, t];
```

Assign these to functions cPsol and cTsol:

```
In[12]:= cPsol = cP /. sol[[1]];
cTsol = cT /. sol[[1]];
```

The solution depends on  $k_a$ ,  $k_b$ , and  $k_e$ . We want to adjust these constants so that  $c_P(t)$  fits the data.

**Fitting the Data:** Construct the least squares functional that depends on  $k_a$ ,  $k_b$ , and  $k_e$ . Omit missing data point for subject 3 at time 4 hours.

```
In[14]:= SS = Sum[Sum[(cPsol[times[[j]] - concdata[[i, j]])^2, {i, 1, 5}], {j, 1, 7}];
SS = SS - (cPsol[times[[6]] - concdata[[3, 6]])^2;
```

Now we need to minimize SS as a function of  $k_a$ ,  $k_b$ , and  $k_e$ . This can be done by setting partial derivatives equal to zero or by using Mathematica's "FindMinimum" command. It may help to provide an initial guess in which  $k_a$ ,  $k_b$ , and  $k_e$  are all positive.