Solving Linear Systems with the Laplace Transform

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This worksheet illustrates using the Laplace transform to solve linear system of ODEs (and also just using Maple's ***dsolve*** command).

> $with(inttrans)$:

Consider the linear constant-coefficient nonhomogeneous system

> $de1 := diff(x1(t), t) = -5 \cdot x1(t) + 6 \cdot x2(t) - 2 \cdot \sin(t) + 10 \cdot \cos(t) - 6 \cdot \exp(t);$
> $de2 := diff(x2(t), t) = -3 \cdot x1(t) + x2(t) + 6 \cdot \cos(t)$

with initial data x1(0) = 2 and x2(0) = 1.

To solve, Laplace transform both sides of both equations

> $de1lap := laplace(de1, t, s);$
> $de2lap := laplace(de2, t, s);$

Substitute in the initial conditions, and (for convenience) let X1 = laplace(x1(t),t,s) and X2 = laplace(x2(t),t,s):

> $de1lap2 := subs(x1(0) = 2, x2(0) = 1, laplace(x1(t), t, s) = X1, laplace(x2(t), t, s) = X2, de1lap);$
> $de2lap2 := subs(x1(0) = 2, x2(0) = 1, laplace(x1(t), t, s) = X1, laplace(x2(t), t, s) = X2, de2lap);$

Solve for the transforms X1 and X2

> $Xsols := solve(\{de1lap2, de2lap2\}, \{X1, X2\})$

Let X1sol and X2sol denote the transforms

> $X1sol := subs(Xsols, X1);$
> $X2sol := subs(Xsols, X2);$

Inverse transform to find the solutions

> $x1sol := invlaplace(X1sol, s, t);$
> $x2sol := invlaplace(X2sol, s, t);$

A quick check using the ***dsolve*** command:

> $dsolve(\{de1, de2, x1(0) = 2, x2(0) = 1\}, \{x1(t), x2(t)\})$

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