Homogeneous Linear Systems of ODEs in Maple

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This worksheet illustrates how to analyze homogeneous linear systems of ODEs in Maple.

First, load in the "plots" and "LinearAlgebra" package

```
> with(plots) :
  with(LinearAlgebra) :
```

Example System: Consider the homogeneous linear system of ODEs

```
> de1 := x1'(t) = x1(t) + 3·x2(t);
  de2 := x2'(t) = 3·x1(t) + x2(t)
```

for functions x1(t) and x2(t), with initial data x1(0) = 2, x2(0) = 6.

Solution via dsolve: The solution can be obtained with Maple's ***dsolve*** command:

```
> sol := dsolve({de1, de2, x1(0) = 2, x2(0) = 6}, {x1(t), x2(t)})
```

Solution via Eigen-analysis: The system above is of the form **x**' = **Ax** where **x** = <x1,x2> and

```
> A := Matrix([[1, 3], [3, 1]])
```

We solve using the eigenvector/value analysis of Section 6.2.2. The eigenvalues and vectors of A can be computed as

```
> eigs := Eigenvectors(A)
```

The first part of "eigs" (the column vector <4,-2>) has as its components the eigenvalues of A, that is, the eigenvalues are

```
> eigs[1][1], eigs[1][2]
```

The second part of "eigs" (the 2 x 2 matrix) has as its columns the corresponding eigenvectors of A. Define a matrix **P**

```
> P := eigs[2]
```

as was done in equation (6.18) in the text. Let

```
> x0 := ⟨2, 6⟩
```

denote the initial condition vector. As per the procedure in Section 6.2.2, we can construct the solution to this ODE system by first solving **Pc = x0** for **c**, which in Maple takes the form

```
> c := LinearSolve(P, x0)
```

From (6.17) in the text the solution is then

```
> xsol := c[1]·exp(eigs[1][1]·t)·Column(P, 1) + c[2]·exp(eigs[1][2]·t)·Column(P, 2)
```

The command "Column(P, j)" picks off the jth column of a matrix "P".

The components x1(t) and x2(t) are given by

```
> xsol[1]; xsol[2];
```

Example 2: Consider the system

```
> de1 := x1'(t) = -5· x1(t) + 6·x2(t);
  de2 := x2'(t) = -3·x1(t) + x2(t);
```

with initial data x1(0) = 1, x2(0) = 2. This system is governed by the matrix

```
> A := Matrix([[-5, 6], [-3, 1]])
```

Compute the eigenvalues/vectors as

```
> eigs := Eigenvectors(A)
```

Following the analysis in the previous example, pick off the eigenvectors and install them in a matrix

P as

> $P := eigs[2]$

In the general solution (6.17) in the text, the desired initial conditions are obtained by taking the c[k] as the components of the vector **c** that satisfies **Pc = x0** where **x0** = <1,2> (equation (6.18)). Compute

> $c := LinearSolve(P, \langle 1, 2 \rangle)$

The solution (from (6.17)) is then

> $xsol := c[1] \cdot \exp(eigs[1][1] \cdot t) \cdot Column(P, 1) + c[2] \cdot \exp(eigs[1][2] \cdot t) \cdot Column(P, 2)$

However, a real-valued version is obtained with

> $evalc(xsol)$

>