PID Control Example

Kurt Bryan and SIMIODE

A notebook to illustrate the incubator P/PI/PID control computations for Section 5.6.

System/Plant Model: The uncontrolled incubator temperature is governed by the Newton cooling ODE y'(t) = -k*(y(t) - a(t)) where "k" is the cooling constant and "a(t)" is ambient temperature. We let us take, for the moment,

In[17]:=
```
k = 0.05
a[t_] = 0
```

The control function will be u(t), and the constant "K" in the controlled ODE (equation (5.108) in the text) will be K = 1. The controlled ODE is thus

In[19]:=
```
K = 1
controlled_ODE = y'[t] == -k*(y[t] - a[t]) + K*u[t]
```

The desired temperature (setpoint) will be 0 degrees for t < 20 and then 3 degrees for t > 20.

In[21]:=
```
r[t_] = 3*UnitStep[t - 20]
```

Assume the initial condition is y(0) = y0 with

In[22]:=
```
y0 = 5
```

The Control: Choose the control gains for PID control. In this case we will use PI control (so Kd = 0):

In[23]:=
```
Kp = 1
Ki = 1/10
Kd = 0
```

The plant transfer function Gp(s) and controller transfer Gc(s) are, from (5.111) and (5.129) respectively,

In[26]:=
```
Gp[s_] = K/(s + k)
Gc[s_] = Kp + Ki/s + Kd*s
```

From (5.118) the closed-loop transfer function is

In[28]:=
```
G[s_] = Gp[s]*Gc[s]/(1 + Gp[s]*Gc[s])
```

The System Response to a Disturbance: This system starts off at the wrong temperature (5 degrees instead of the desired setpoint 0) and there is a disturbance in the form of an abrupt setpoint change at time t = 20.

The governing ODE is y'(t) = -k(y(t) -a(t)) + K*u(t) where u(t) = r(t) - y(t). According to equation (5.133) the system response in the s domain can be computed as Y(s) = G(s)*R(s) + y0*Gp(s)/(1+Gp(s)*Gc(s)).

Using Mathematica

In[29]:=
```
R = LaplaceTransform [r[t], t, s]
Y = Simplify [G[s] * R + y0 * Gp[s] / (1 + Gc[s] * Gp[s])]
```

The time domain response is then

In[31]:=
```
ysol = InverseLaplaceTransform [Y, s, t]
```

A plot:

In[35]:=
```
Plot[{r[t], ysol}, {t, 0, 100}, PlotStyle → {Red, Blue}]
```

Note the system starts off at 5 degrees (where the desired setpoint is zero), so the control cools the system to 0 degrees, with a bit of overshoot. The controller effectively responds to the change in the setpoint at time t = 20.