# PID Control Example

**Kurt Bryan and SIMIODE**

A worksheet to illustrate the incubator P/PI/PID control computations for Section 5.6.

**System/Plant Model:** The uncontrolled incubator temperature is governed by the Newton cooling ODE y'(t) = -k*(y(t) - a(t)) where "k" is the cooling constant and "a(t)" is ambient temperature. We let us take, for the moment,

```
k = 0.05
syms t;
a = @(t) 0; %Ambient temperature a(t) is the zero function
```

The control function will be u(t), and the constant "K" in the controlled ODE (equation (5.108) in the text) will be K = 1. The controlled ODE is thus

```
K = 1;
syms y(t) u(t);
controlled_ODE = diff(y,t)==-k*(y(t)-a(t))+K*u(t)
```

The desired temperature (setpoint) will be 0 degrees for t < 20 and then 3 degrees for t > 20.

```
r(t) = 3*heaviside(t-20)
```

Assume the initial condition is y(0) = y0 with

```
y0 = 5
```

The Control: Choose the control gains for PID control. In this case we will use PI control (so Kd = 0):

```
Kp = 1; %Proportional gain
Ki = 1/10; %Integral gain
Kd = 0; %Derivative gain
```

The plant transfer function Gp(s) and controller transfer Gc(s) are, from (5.111) and (5.129) respectively,

```
syms Gp(s); Gp(s) = K/(s+k);
syms Gc(s); Gc(s) = Kp + Ki/s + Kd*s;
```

From (5.118) the closed-loop transfer function is

```
G(s) = Gp(s)*Gc(s)/(1+Gp(s)*Gc(s))
```

**The System Response to a Disturbance:** This system starts off at the wrong temperature (5 degrees instead of the desired setpoint 0) and there is a disturbance in the form of an abrupt setpoint change at time t = 20.

The governing ODE is y'(t) = -k(y(t) -a(t)) + K*u(t) where u(t) = r(t)  - y(t). According to equation (5.133) the system response in the s domain can be computed as Y(s) = G(s)*R(s) + y0*Gp(s)/(1+Gp(s)*Gc(s)). Using Matlab

```
R = laplace(r(t),t,s);
Y = simplify(G(s)*R + Gp(s)*y0/(1+Gp(s)*Gc(s)))
```

The time domain response is then

```
ysol = ilaplace(Y,s,t)
```

A plot

```
fplot([r(t),ysol],[0 100])
```

Note the system starts off at 5 degrees (where the desired setpoint is zero), so the control cools the system to 0 degrees, with a bit of overshoot. The controller effectively responds to the change in the setpoint at time t = 20.