Fish Harvesting Revisited

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Notebook for fish logistic-growth harvesting project in Section 3.5.2.

The Data: The population data from Table 3.10, starting with 1978, which we will call year 0.

In[19]:=
```
udata = {72148, 73793, 74082, 92912, 82323, 59073, 59920, 48789, 70638, 67462,
  68702, 61191, 49599, 46266, 34877, 28827, 21980, 17463, 18057, 22681,
  20196, 25776, 23796, 19240, 16495, 12167, 21104, 18871, 21241, 22962}
```

However, Mathematica indexes from 1 to 30, so udata[[1]] = 72148.

The harvesting data is

In[20]:=
```
hd = {0.18847, 0.149741, 0.21921, 0.17678, 0.28203, 0.34528, 0.20655,
  0.33819, 0.14724, 0.19757, 0.23154, 0.20860, 0.33565, 0.29534, 0.33185,
  0.35039, 0.28270, 0.19928, 0.18781, 0.19357, 0.18953, 0.17011, 0.15660,
  0.28179, 0.25287, 0.25542, 0.08103, 0.087397, 0.081952, 0.10518}
```

A plot of each data set:

In[21]:=
```
ListPlot[udata]
```

In[22]:=
```
ListPlot[hd]
```

Fitting the Data to the ODE: Form finite difference approximations (u(k+1) - u(k))/1 from the population data, for k = 1 to k = 29 (= n-1). This approximates u'(t) when t = k:

In[23]:=
```
udiffdata = Table[udata〚k + 1〛 - udata〚k〛, {k, 1, 29}]
```

Next substitute u = udata[[k]] into the harvested logistic ODE u'(t) = r*u(t)*(1-u(t)/K) - h(t)*u(t) for k = 0 to k = n-1, to approximate the right side of the ODE at time t = k for k = 0 to k = n-1. This expression depends on r and K.

In[24]:=
```
udiffdata2 = Table[r * udata〚k〛 * (1 - udata〚k〛 / K) - udata〚k〛 * hd〚k〛, {k, 1, 29}];
```

Form a sum of squares that depends on r and K

In[25]:=
```
SS = Sum[(udiffdata〚k〛 - udiffdata2〚k〛)^2, {k, 1, 29}];
```

Finding the optimal r and K: Start with a plot of SS(r, K), or log(SS).

In[26]:=
```
Plot3D[Log[SS], {K, 40000, 300000}, {r, 0.1, 0.5}]
```

Or perhaps a contour plot:

In[27]:=
```
ContourPlot[Log[SS], {K, 40000, 300000}, {r, 0.1, 0.5}, Contours → 50]
```

Something near r = 0.3, K = 200000 might be a good initial guess at a minimizer for SS.

If we use these values for r and K (but you should find the true minimizer) we could compare the predicted cod population to the data by following the suggestion of Modeling Exercise 5.2.3. Let U be an array to hold our numerical estimates of the cod population:

In[28]:=
```
U = Table[0, {k, 1, 30}];
```

Now use the harvested logistic ODE with our estimates for r and K to march the predicted cod biomass out in time (this is essentially Euler's method):

In[29]:=
```
U⟦1⟧ = udata⟦1⟧
For[k = 1, k ≤ 29, k++,
U⟦k + 1⟧ = (U⟦k⟧ + r * U⟦k⟧ * (1 - U⟦k⟧ / K) - hd⟦k⟧ * U⟦k⟧) /. {r → 0.3, K → 200 000}
]
```

In[31]:=
```
p1 = ListPlot[udata];
p2 = ListPlot[U, PlotStyle → {Red}];
Show[p1, p2]
```