

# Direction Fields

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A script to illustrate how to draw a direction field for an ODE, and superimpose a solution curve.

We'll use  $u'(t) = t \cos(u(t)) - \sin(t)$  as an example. Define the right side  $t \cos(u) + \sin(t)$  as a function of  $t$  and  $u$ :

```
f = @(t,u) t.*cos(u)-sin(t)
```

Now construct the direction field on the range  $0 \leq t \leq 5$ ,  $0 \leq u \leq 5$ . First

```
[t,u] = meshgrid(0:0.25:5, 0:0.25:5);
```

sets up a grid in the  $(t,u)$  plane on the given region with 0.25 spacing between grid points in each direction. Then compute the slope "s" at each grid point with

```
s = f(t,u);
```

Finally, have Matlab plot arrows at the appropriate grid points, with the appropriate slopes. We can scale the arrows to have an appealing length using a final argument "2" (you can choose it as you wish):

```
plot1 = quiver(t,u,ones(size(s)), s, 2);  
axis tight
```

To superimpose solution curves, solve the ODE numerically. For example, to show a solution curve with initial data  $u(1) = 2$ , call Matlab's solver

```
inittime = 1.0; ic = 2.0; finalT = 5.0;  
[tm,um] = ode45(f,[inittime,finalT], ic); %Matlab's solver  
hold on;  
plot2 = plot(tm,um,'-r');  
hold off
```

Alternatively, use the supplied subroutine "draw\_dirfield" in the form `draw_dirfield(f, range ,ics)` where "range" is of the form `[lowt, hight, lowu, highu]` and `ics` is an optional  $n \times 2$  matrix of initial conditions with  $k$ th row  $t_k$ ,  $u_k$  (where  $u(t_k) = u_k$ .)

```
ics = [0 1;1 2;1 4];  
draw_dirfield(f,[0,5,0,5],ics)
```