1. Below are two vectors \( \mathbf{v} \) and \( \mathbf{w} \). Sketch a good picture of both \( \mathbf{v} + \mathbf{w} \) and \( \mathbf{v} - \mathbf{w} \), below the figure somewhere.

   **Solution:** See below

   ![Figure 1: Pretty vectors added and subtracted.](image)

2. Find a vector that points in the direction of the vector \(<3, 4>\) and has length 8.

   **Solution:** The vector \(<3, 4>\) has magnitude \(\sqrt{3^2 + 4^2} = 5\), so \(<3/5, 4/5>\) is a unit vector parallel to \(<3, 4>\). Then \(8 <3/5, 4/5> = <24/5, 32/5>\) has length 8 and is parallel to \(<3, 4>\).

3. If \( \mathbf{u} = <1, 2, 3> \) and \( \mathbf{v} = <1, -1, 1> \), find \(|\mathbf{u}|, |\mathbf{v}|, \mathbf{u} \cdot \mathbf{v}\), and the cosine of the angle between \( \mathbf{u} \) and \( \mathbf{v} \).

   **Solution:** \(|\mathbf{u}| = \sqrt{14}, |\mathbf{v}| = \sqrt{3}, \mathbf{u} \cdot \mathbf{v} = 2\), and \(\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|} = \frac{2}{\sqrt{42}}\).

4. (Bonus) In the figure below the point \( C = (\cos(\theta), \sin(\theta)) \) for some \( \theta \) (the circle is the unit circle centered at the origin). Show that the segments \( \overline{CA} \) and \( \overline{CB} \) are orthogonal. Hint: think of them both as vectors. Explain your reasoning!

   **Solution:** The segment \( \overline{AC} \) can be considered as a displacement vector \( \mathbf{v} \), say \( \mathbf{v} = <\cos(\theta) + 1, \sin(\theta)> \) (if we put the tail at \( A \)). Similar \( \overline{AB} \) can be thought of as \( \mathbf{w} = <\cos(\theta) - 1, \sin(\theta)> \). Then

   \[
   \mathbf{v} \cdot \mathbf{w} = (\cos(\theta) - 1)(\cos(\theta) + 1) + \sin^2(\theta) = \cos^2(\theta) - 1 + \sin^2(\theta) = 0.
   \]

   So the segments/vectors are orthogonal.
Figure 2: Unit circle and inscribed angle.