

Prime Time!

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1. A whole number is said to be *prime* if it cannot be divided by any other whole number, except itself and 1. (Every whole number is divisible by itself and 1 anyway). Fractions are not allowed!

For example, 5 is prime. You can divide 5 by itself ($5 \div 5 = 1$) and you can divide 5 by 1 ($5 \div 1 = 5$), but 5 is not divisible by any other whole number, since

$$5 \div 2 = 2\frac{1}{2}, \quad 5 \div 3 = 1\frac{2}{3}, \quad 5 \div 4 = 1\frac{1}{4}.$$

And isn't it pretty obvious that 5 can't be divided by any number larger than 5?

→ Check the number 7 to see if it's prime—can it be divided by 2, 3, 4, 5, or 6, or anything else besides itself and 1?

2. Here's a fact: the number 1 is NOT considered to be a prime. The number 2 is the “first” prime number, followed by 3, then 5, then 7, and so on.
3. If a number is not prime, it is said to be *composite*. A number is therefore composite if it can be divided by some other whole number, except for 1 and itself. For example, 6 is composite because 6 is divisible by 2 ($6 \div 2 = 3$). Of course, 6 is also divisible by 3.

→ Check that the number 21 is composite—what whole number(s) divide 21 cleanly?

4. Trying to decide if a number is prime or composite can take quite a bit of work! Spend a few minutes and try to decide if the number I give you is prime or composite. Write your number in the space below. If it is prime, write “is prime” after the number. If it is composite, write “is composite” after the number, AND write down the whole number that divides your number.
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5. Suppose we want to generate a list of all the prime numbers up to 100 (or maybe even larger). Testing each number one at a time would take a HUGE amount of work. But there's a very clever way to do it!

(a) Start with a list of all the numbers from 1 to 100. We're going to cross out most of them—all the composites. What will be left are the primes. Take the list I give you and cross out 1, since it's not considered to be a prime.

- (b) The next number in the list (2) is prime, so circle it. Now go through the list and cross out every multiple of 2 (except 2 itself). That is, cross out 4, 6, 8, 10, . . . , 100, since all of them are divisible by 2 and so not prime.
- (c) The next number in the list (3) is prime, so circle it. Now go through the list and cross out every multiple of 3 (except 3 itself). That is, cross out 6, 9, 12, 15, . . . , 99, since all of them are divisible by 3 and so not prime. Notice some are already crossed out, because they were divisible by 2.
- (d) The next number in the list (5) is prime, so circle it. Now go through the list and cross out every multiple of 5 (except 5 itself). That is, cross out 10, 15, 20, 25, . . . , 100, since all of them are divisible by 5 and so not prime. Notice some are already crossed out, because they were divisible by 2 or 3.
- (e) Repeat this process of finding the next number in the list and then crossing out all multiples of it, until you run out of numbers. What's left are the primes!

This process of generating primes is called the *Sieve of Eratosthenes*. (Eratosthenes was a Greek mathematician who lived from 276 BC to 194 BC. He was the first person to accurately measure the size of the earth.)

- 6. Is there a largest prime number? NO! The Greek mathematician Euclid proved 2300 years ago that no matter how large a prime you find, there is another prime which is larger. The supply of prime numbers is inexhaustible!
- 7. Pick an even number between 20 and 100. Find as many ways as you can to write that number as a sum of two primes; use the list of primes we constructed to help you! How many ways can you write your even number as a sum of two primes?
- 8. Start with the number I give you. Divide it by 2, if possible. Take what's left (the "quotient") and try dividing it by 2 again, and repeat until you can no longer divide it by 2. Keep a record of how many times you were able to divide by 2.

It is believed that every even number (bigger than 2) can be written as a sum of two primes. Using computers, mathematicians have checked every even number up to 300,000,000,000,000,000! But no one has proved that it will always work.

Now take what's left and try dividing it by 3, over and over, until you can't divide by 3 anymore; keep track of many times you can divide by 3. Then do this with 5, 7, 11, and all the others primes (use your list of primes). Keep going until the number you have left is 1.

You've now "factored" the original number, that is, figured out how to write it as a product of prime numbers. Write your factorization below: